# DMT COMPANION

Insight 47 · 2026 · for instructional coaches

Math Success by DMTI

## RATIO TABLES: AN INSTRUCTIONAL COACHING COMPANION

*Using student thinking to develop multiplicative reasoning*

### How to Use This

This Companion is for instructional coaches working with DMT Insight 47. The Insight reviews the research on multiplicative reasoning, unitizing, and representation. This Companion focuses on what teachers and coaches can notice in student work, questions for coaching conversations, a five-phase instructional sequence, and a three-day lesson sequence with coach look-fors.

> **Key Idea • Students need to understand the unit relationship and be able to reason back to it.**

### Three Guiding Questions

*Bring these to a coaching conversation or a PLC.*

1. **What relationship is the student using?**
2. **Where is the unit visible in the student's work?**
3. **Is the student iterating, composing, decomposing, or scaling?**

### The Unit Relationship

Every column in a ratio table records one multiplicative relationship. Early on, writing the column for 1 makes that relationship explicit. Later, students reason from any known pair. A student who knows that 4 corresponds to 20 can reason to 2 and 10, 1 and 5, 8 and 40, or 12 and 60. Requiring a written column for 1 in every table can become another procedure.

> **Key Idea • Return to the unit relationship when students lose the multiplicative structure.**

## What to Look For in Student Work

Two tables can look alike and record different reasoning. In the first table, the student keeps a difference from one column to the next. In the second, the student keeps a factor.

**A student's table**

| Bags | 1 | 2 | 3 |
|---|---|---|---|
| Apples | 4 | 6 | 8 |

+ 2 + 2

*keeps a difference of 2*

**The multiplicative relationship**

| Bags | 1 | 2 | 3 |
|---|---|---|---|
| Apples | 4 | 8 | 12 |

1 × 4 2 × 4 3 × 4

*keeps a factor of 4*

*Left: an additive response to 4 apples in each bag. Right: the multiplicative relationship.*

**Questions to bring to student work:**

- Is the student reasoning additively or multiplicatively? What is the unit?
- Is the student iterating one unit at a time? Can the student compose known quantities?
- Can the student decompose a target quantity? Can the student explain why two columns are equivalent?
- Can the student move backward to determine the unit? Can the student transfer the relationship to a new representation?

## What Student Work Can Reveal

*Not everything in a student's table is a misconception. Each row offers a possible interpretation, not a diagnosis.*

| What You See | Possible Interpretation | Coaching or Teacher Response |
|---|---|---|
| 1→4, 2→6, 3→8 | The student is preserving an additive difference rather than a multiplicative relationship. | Ask: "What stays the same from one column to the next?" Compare two columns with the unit relationship. |
| Builds from a known column and does not write a column for 1 | The student may be reasoning from a known pair. Check whether the student can reason back to the unit. | Ask: "What does 1 correspond to? How do you know?" A written column for 1 is not required if the student can answer. |
| Counts sequentially to 37 | The student understands iteration but is not yet composing units efficiently. | Ask: "What if we jumped to 10? To 30? What would you still need?" |
| Several columns are incorrect after one early error | The student builds each column from the one before it and does not check against a known pair. | Return to the unit relationship. Rebuild one column at a time and check each against a known pair. |

## Questions for Coaching Conversations

**Questions to ask teachers:**

- What relationship is the student using?
- What does the student appear to think stays the same?
- What does each row represent? What does each column represent?

- Where is the unit visible?
- What known relationship could the student use?
- Is the student iterating, composing, decomposing, or scaling?
- What would you ask next without telling the student the procedure?

**A question to use in a PLC:**

Give teachers a student ratio table with an additive error, such as 1→4, 2→6, 3→8. Ask: "What does this student appear to believe stays the same?" Then compare it with a table that keeps the multiplicative relationship.

> **Key Idea • Identify the relationship the student is using before deciding what to ask next.**

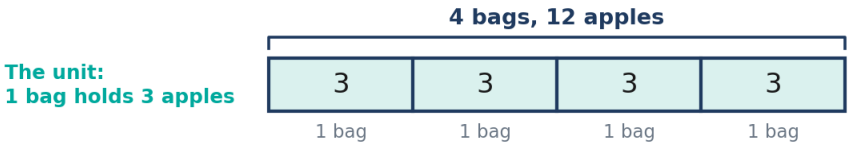
## The Instructional Sequence: Five Phases

The five phases describe a DMTI instructional approach for Grades 3-6. Use them to plan with teachers and to locate where a class is working. Students move back and forth among the phases as the numbers and contexts change.

### Phase 1 · make the unit relationship visible.

Goal: use a bar model or double number line to establish the unit and the multiplicative relationship. Connect the model to the table so that students see the same relationship in both.

Task: 1 bag holds 3 apples. Draw 4 bags with a bar model.




Questions: "What is one unit?" "How many units are there?" "How are the units alike?"

### Phase 2 · record the relationship in a ratio table

Goal: record the relationship from the bar model in a table. Begin with the unit relationship and build several sequential values.

Each column: bags × 3

| Bags | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Apples | 3 | 6 | 9 | 12 |

+ 3 + 3 + 3

Building several sequential values helps students establish and verify the unit relationship before making larger jumps.

Questions: "What does this column tell us?" "How does each column relate to the unit relationship?"

### Phase 3 · reason from known relationships

Goal: move from iterating one unit at a time to reasoning from known pairs.

Questions: “Do we have to go 1, 2, 3, 4? What if we need 10?” “What if we need 7? Could you decompose it into quantities you already know?”

| Bags | 1 | 5 | 10 | 7 |
|---|---|---|---|---|
| Apples | 3 | 15 | 30 | ? |

**Two ways to build 7**

$7 = 5 + 2$

$7 = 10 - 3$

Students use strategies such as $7 = 5 + 2$ or $7 = 10 - 3$ with columns they have already built. Students can build the same quantity from different known relationships.

### Phase 4 · compose and decompose.

Goal: develop efficiency by choosing which quantities to compose and decompose.

Task: If 1 corresponds to 4, find $13 \times 4$. Decompose 13 fours into 10 fours and 3 fours. Then 40 and 12 compose 52.

| Units of 4 | 1 | 10 | 3 | 13 |
|---|---|---|---|---|
| Total | 4 | 40 | 12 | 52 |

$13 = 10 + 3$

$52 = 40 + 12$

Questions: “How did you decompose 13?” “Which numbers are easier to use?” “How could someone solve it differently and get the same answer?”

### Phase 5 · scale and generalize

Goal: reason in both directions and connect the table to unit rate, proportion, and linear functions.

Task: If 4 corresponds to 20, what corresponds to 1? What corresponds to 8? Students divide to find the unit relationship, then scale up. When students graph the pairs with the number of boxes on the horizontal axis, the points lie on a line through the origin. The slope of that line represents the constant of proportionality.

**Known:**
**4 boxes hold 20 pencils**

| | | ÷ 4 | × 2 |
|---|---|---|---|
| Boxes | 1 | 4 | 8 |
| Pencils | 5 | 20 | 40 |
| | | ÷ 4 | × 2 |

Questions: “How did you get back to 1?” “What stays the same in every column?” “What would you see if you graphed these pairs?”

> **Key Idea • The five phases are a planning tool, not a rigid sequence.**

## Teacher Moves to Look For

| | |
|---|---|
| Return to the unit relationship | When a student's table breaks down, the teacher asks, "What does 1 represent?" and rebuilds from a known pair. |
| Ask for the relationship. | After an answer, the teacher asks how the student built, composed, decomposed, or scaled the quantities: "How did you build that?" "What did you compose?" "What relationship did you use?" |
| Compare strategies | The teacher records two or three strategies side by side and asks what they share and which is more efficient. |
| Connect procedures to relationships. | Cross multiplication follows students' reasoning about equivalent ratios and is connected to that reasoning. |

## A Sample Three-Day Lesson Sequence

Each day follows a launch, explore, discuss, and close structure. The sequence works for Grades 3-6. A coach look-for follows each day.

| | |
|---|---|
| Day 1<br>From Visual to Table | Goal: Build a ratio table for a simple multiplicative context; explain the unit.<br>• Launch (5 min): Pose an equal-units context. Students draw a bar model.<br>• Explore (15 min): Students record the columns for 1 to 4 and connect each column to the bar model.<br>• Discuss (10 min): "What is the unit? How does every column relate to it?"<br>• Close (5 min): Students write one sentence about what one column means in context.<br>**Coach look-for:** Can students identify the unit and explain what one column represents? |
| Day 2<br>Building Flexibility | Goal: Use scaling (×2, ×5, ×10) and decomposition to extend a table efficiently.<br>• Launch (5 min): Review a Day 1 table. "What if I need 10? How can I find 10 without counting up?"<br>• Explore (15 min): Students scale known pairs and compose values such as 7, 13, and 23.<br>• Discuss (10 min): Share two or three strategies for the same target. "Did anyone decompose it differently?"<br>• Close (5 min): Exit task. Students solve one missing-value problem by reasoning from known quantities rather than building every value sequentially.<br>**Coach look-for:** Do students begin composing known quantities rather than iterating every unit? |
| Day 3<br>Generalization and Connection | Goal: Apply flexible reasoning to a new context; begin connecting to rate.<br>• Launch (5 min): Pose a new context, such as cost per item or distance per hour. Students identify the unit relationship.<br>• Explore (15 min): Students build the table flexibly and compose in more than one way.<br>• Discuss (10 min): "What stays the same in every column? What would happen if you graphed these pairs?"<br>• Close (5 min): Students describe the unit relationship in their own words.<br>**Coach look-for:** Can students maintain the same multiplicative relationship in a new context? |

## What Progress Looks Like

Over time, students:

- Identify the unit relationship and reason back to it from any known pair
- Compose and decompose known quantities rather than relying only on sequential iteration
- Explain why a column is correct by relating it to another column
- Maintain the same multiplicative relationship across every column
- Use the same relationships across contexts: equal units, price, distance, rate

## Practice Resources from DMTI

Seven practice decks develop ratio-table fluency across Grades 3-6. Coaches can use the progression to plan with teachers. The decks are available at www.dmtinstitute.com and www.teacherspayteachers.com (search DMTI, Ratio Tables).

| | Title | What It Builds | Standards |
|---|---|---|---|
| A | Doubling | First multiplicative structure. Entry point for the whole series. | Gr. 3–6 \| 3.OA.B.5 · 3.OA.C.7 · 3.OA.D.9 · 4.OA.C.5 · 4.NBT.B.5 |
| B | Multiples in Order | Sequential multiples, digit patterns, odd/even products. Builds fluency before flexible use. | Gr. 3–6 \| 3.OA.B.5 · 3.OA.C.7 · 3.OA.D.9 · 4.OA.C.5 · 4.NBT.B.5 |
| C | Composing Multiples | Use a completed table to solve larger problems by composing columns. | Gr. 3–6 \| 3.OA.C.7 · 3.OA.D.9 · 4.NBT.B.5 · 5.NBT.B.5 · 6.RP.A.3 |
| D | 1–2–5–10 Table | Build four landmark columns and compose other products from them. Develops flexible composition and decomposition within the ratio table. | Gr. 4–6 \| 4.OA.C.5 · 4.NBT.B.5 · 5.NBT.B.5 · 5.NBT.B.6–7 · 6.RP.A.3 |
| E | Powers of 10 (Whole #s) | Apply the ratio table to powers of 10 with whole numbers: scaling by 10, 100, and 1,000. | Gr. 4–6 \| 4.NBT.B.5 · 5.NBT.B.2 · 5.NBT.B.5 · 5.NBT.B.6 · 6.RP.A.3 |
| F | Powers of 10 (Decimals) | The multiplicative reasoning from Deck E, extended to decimals. | Gr. 5–6 \| 5.NBT.B.2 · 5.NBT.B.5 · 5.NBT.B.6–7 · 6.RP.A.3 |
| G | Division | Division as scaling down, with the structure of Decks E and F in the other direction. | Gr. 5–6 \| 5.NBT.B.2 · 5.NBT.B.6–7 · 6.NS.B.2 · 6.RP.A.3 |

Decks A and B address Grade 3 content. Decks C and D develop flexible composition and decomposition. Decks E and F extend to powers of 10. Deck G treats division as scaling down. Each deck uses the same structure, so the same multiplicative relationships extend as the numbers and contexts become more complex.

## Where to Go Next

Read DMT Insight 47 for the research behind this Companion. Find it at mathsuccess.io/dmtinsights.