

DMT COMPANION

Insight 43 · 2026 · for teachers, coaches, and school leaders



SUMMER MATHEMATICS: DEVELOPING UNDERSTANDING, LANGUAGE, AND FLUENCY

Planning summer instruction that connects quantities, language, and practice

How to Use This

This Companion is for teachers, coaches, and school leaders planning or teaching a summer mathematics program with DMT Insight 43. It summarizes the research, provides examples that connect story mats, models, mathematical language, and practice, and suggests ways to evaluate student learning.

Key Idea • Summer instruction provides time to develop conceptual understanding and procedural fluency together. Students identify units and quantities, represent mathematical relationships, explain their reasoning, and practice for accuracy, efficiency, and flexibility.

Three Guiding Questions

Consider these questions as you plan your summer program and examine student work.

1. Which mathematical relationships do your students most need to develop this summer?
2. Where in each session do students use mathematical language to explain quantities and relationships?
3. How will you know what students learned in mathematics, and in mathematical language?

What the Research Shows

Each study measured something different. None evaluated DMTI's program or story mats.

Study	Students	Reported result	What it measured
Lynch, An, & Mancenido (2023)	37 studies, pre-K to Grade 12	Average positive effect of 0.10 standard deviations on mathematics achievement; similar in higher- and lower-poverty settings	Mathematics achievement
Snipes et al. (2015). Elevate Math	Rising Grade 8, 19 days	Effect of 0.70 standard deviations on algebra readiness; 29% versus 12% met the specified threshold	Algebra readiness
Lynch & Kim (2017).	Grades 3–9, online program	More reported home engagement; similar achievement	Engagement and achievement
Hassinger-Das et al. (2015).	Kindergarten, 124 students	Gains in mathematical vocabulary; no statistically significant improvement on the general mathematics measure	Mathematical vocabulary
Purpura et al. (2017).	Preschool	Gains in mathematical language and early numeracy	Mathematical language and numeracy

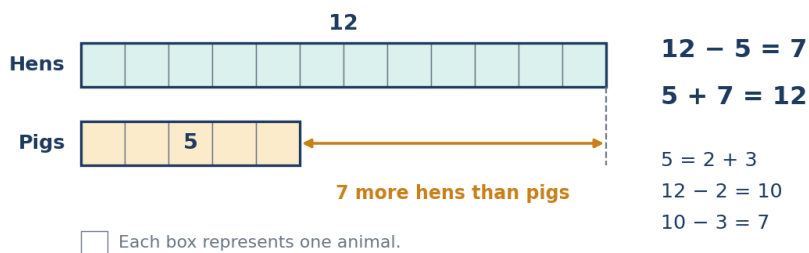
Worked Examples

Example 1 • the farm story mat: comparing 12 hens and 5 pigs

On DMTI's farm story mat, students count 12 hens in the yard and 5 pigs in the mud. The teacher asks, "How many more hens are there than pigs?" Students identify the two quantities being compared, represent their relationship with a bar model, and determine that there are 7 more hens than pigs. They might decompose 5 into 2 and 3, subtract 2 from 12 to make 10, and then subtract 3 more: $12 - 5 = 7$. They can also explain the relationship as $5 + 7 = 12$.



DMTI's farm story mat.



"There are 12 hens and 5 pigs. There are 7 more hens than pigs."

A bar model comparing 12 hens and 5 pigs. Each box represents one animal: $12 - 5 = 7$.

Questions to ask:

"What does the 12 represent? What does the 5 represent?"

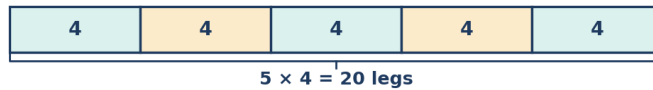
"Where is the difference in your bar model? How many more hens are there?"

"How does $5 + 7 = 12$ describe the same relationship as $12 - 5 = 7$?"

Example 2 • equal groups on the farm: the legs of 5 pigs

Each pig on the story mat has 4 legs. To find how many legs the 5 pigs have, students identify a group of 4 legs as the unit being iterated. Five equal groups of 4 legs represent $5 \times 4 = 20$ legs. A bar model records the group size, the number of groups, and the total, and decomposing 5×4 into $(2 \times 4) + (3 \times 4)$ connects the model to an efficient strategy. Practice with related facts then builds accuracy and efficiency.

One group: 4 legs Number of groups: 5 pigs Total: 20 legs



$$5 \times 4 = 20$$

$$(2 \times 4) + (3 \times 4) \\ = 8 + 12 = 20$$

"Four legs make one group. I iterated that unit five times."

Five equal groups of 4 legs: $5 \times 4 = 20 \text{ legs}$.

Questions to ask:

"What is the unit? How many times is it iterated?"

"Where are 2×4 and 3×4 in the bar model?"

"If there were 3 pigs instead of 5, how many legs would there be? How does your model show this?"

Example 3 • teaching mathematical language explicitly

Introduce each term with a model, use it in teacher explanations, and ask students to use it when they explain their own work.

Term	What it means	A student might say
unit	A specified quantity treated as one for counting, measuring, or iterating	"Four legs make one group. I iterated that unit five times."
compose	Combine quantities to form another quantity	"I composed 5 and 7 to make 12."
decompose	Express a quantity as related parts	"I decomposed 5 into 2 and 3."
iterate	Repeat a unit a specified number of times	"I iterated the group of 4 legs five times."
partition	Divide a quantity or unit into parts; specify equal parts when required	"I partitioned 12 into 3 equal groups of 4."
equal	Having the same value or quantity	"The expressions $12 - 5$ and $10 - 3$ are equal because they both represent 7."

Key Idea • Mathematical language is connected to the quantities and operations students are learning. Ask students to use the terms while they explain their models.

Questions That Extend Mathematical Reasoning

Questions about answers and procedures are important. Additional questions can help students explain the quantities and mathematical relationships involved.

Instead of only	Also ask
"What is $12 - 5$?"	"How could you use 10 to find $12 - 5$?"
"How many legs?"	"What is the unit? How many times did you iterate it?"
"Draw a picture."	"Draw a model that shows how the quantities are related."
"Is that right?"	"How does your model show that your answer makes sense?"

Across the Grades

The same approach applies across grade levels—the mathematics changes.

Grade band	Story and model	Language to use	Practice
Grades K–2	12 hens and 5 pigs on the farm story mat; bar model	compare, decompose, equal	Subtraction facts that use 10
Grades 3–5	5 pigs with 4 legs each; bar model or array	unit, iterate, partition	Related multiplication facts
Grades 6–8	In a separate farm situation, there are 2 cows for every 3 cats. Represent equivalent ratios using a table	unit, ratio, equivalent, iterate	Scaling with equivalent ratios

Evaluating What Students Learn

Plan the outcomes you will measure before the program begins. Measure each one separately.

- Mathematics achievement: a pre- and post-program assessment of the mathematics taught. Pre- and post-program assessments can document changes in performance; determining whether the program caused those changes requires an appropriate comparison or evaluation design.
- Mathematical language: students' use of terms such as unit, compose, and iterate during instruction, which is different from performance on an independently assessed mathematical-language measure.
- General language or reading: a separate assessment, if the program intends to influence these outcomes.
- Engagement: participation, discussion, and attitudes, recorded separately from achievement.

For Coaches

Coaches can plan sessions with teachers, observe how students use story mats, models, and mathematical language, and examine student work to decide what instruction should follow.

Questions for a coaching conversation:

- Which quantities and units did students identify, and how did they represent them?
- What mathematical language did students use while explaining their work?
- Where did practice build accuracy and efficiency with the relationships students had developed?
- What will the next session do to develop the relationships students still need?

For School Leaders

School leaders can support summer mathematics instruction by providing sufficient instructional time, appropriate materials, and opportunities for teachers to plan together. They can also review mathematics achievement, mathematical language, and engagement outcomes to determine what students learned and what to adjust in future programs.

Read DMT Insight 43 at mathsuccess.io/dmtinsights.