# DMT COMPANION

Insight 41 · 2026 · for teachers, coaches, principals, and families

## REASONING ABOUT SHAPES: PROPERTIES AND RELATIONSHIPS

*Examining the properties that define shapes and their categories*

### How to Use This

This Companion is for anyone working with DMT Insight 41. The Insight reviews research on prototypes, concept images, and geometric classification. Use the examples and questions here to examine how students classify shapes and to help them reason from properties.

> **Key Idea • A figure is classified by its defining properties. "Diamond" is not a separate mathematical shape, and rotating a square does not change it into something else.**

### Three Guiding Questions

*Use these questions for discussion with teachers, coaches, and administrators.*

1. **When a shape is rotated, do your students keep the same name? How do they explain it?**
2. **Which properties do your students use to decide whether a figure is a triangle, a rectangle, or a square?**
3. **Can your students explain why a square is both a rectangle and a rhombus?**

### Why Students Classify Shapes by Appearance

Students encounter shapes in different positions and contexts. When classroom examples repeatedly show squares resting on a horizontal side and rectangles as long, horizontal figures, students may begin to associate those appearances with the names.

A square rotated onto a vertex is still a square. Calling it a "diamond" does not describe its mathematical properties. Students should learn to identify shapes by examining their sides, vertices, angles, and relationships.

Property-based classification asks a different question: What must be true of every member of this category? A figure is a square because it has four congruent sides and four right angles, no matter the orientation.

## Investigating Sides, Vertices, and Relationships

Polygons are closed plane figures formed by straight line segments:

- The sides are straight line segments, and the figure is closed.
- The segments meet at vertices.
- The number of sides and vertices identifies the polygon family: triangle, quadrilateral, pentagon, hexagon.
- Side lengths, angle measures, and parallel or perpendicular sides distinguish categories within a family.
- Extending sides as lines can help students examine parallel and perpendicular relationships.

Straightness, closure, and segment arrangement are also properties. They determine whether a figure is a polygon; other properties determine which polygon it is.

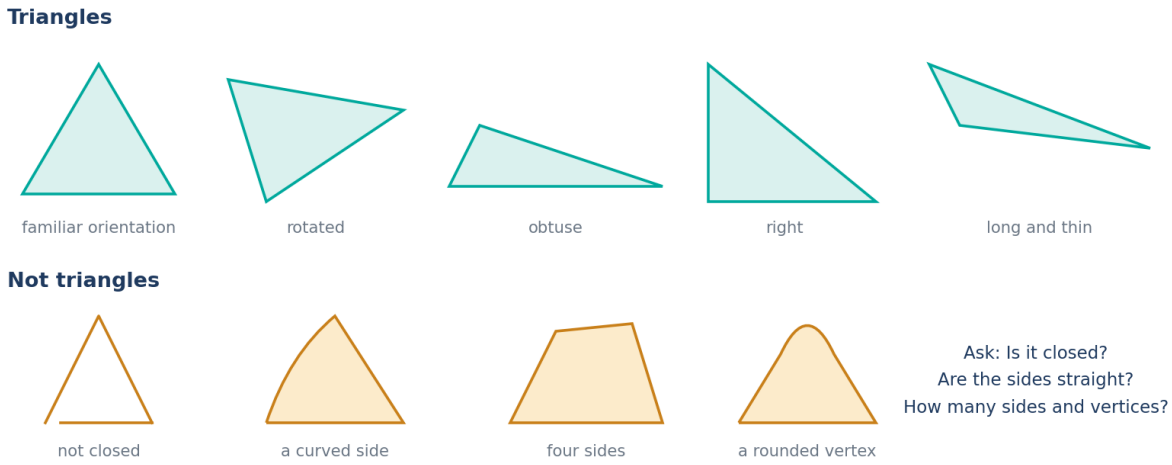



*Figure 1. Examples and nonexamples of triangles in different orientations and proportions.*

## The Same Square, Rotated

Show a square, then rotate it. Ask: “What changed when we rotated the square? What stayed the same?”




**Square**
4 congruent sides, 4 right angles

**The same square, rotated**
4 congruent sides, 4 right angles

**What changed? Its orientation.** **What stayed the same? Every defining property.**

*Figure 2. The rotation changes the square’s orientation, not its properties.*

The rotation changes the square's orientation. Its four congruent sides, four right angles, and two pairs of parallel sides remain unchanged. It is still a square, and it is still a rectangle and a rhombus.

A square belongs to several quadrilateral categories. It is a rectangle because it has four right angles, and it is a rhombus because it has four congruent sides. It is also a parallelogram because both pairs of opposite sides are parallel.

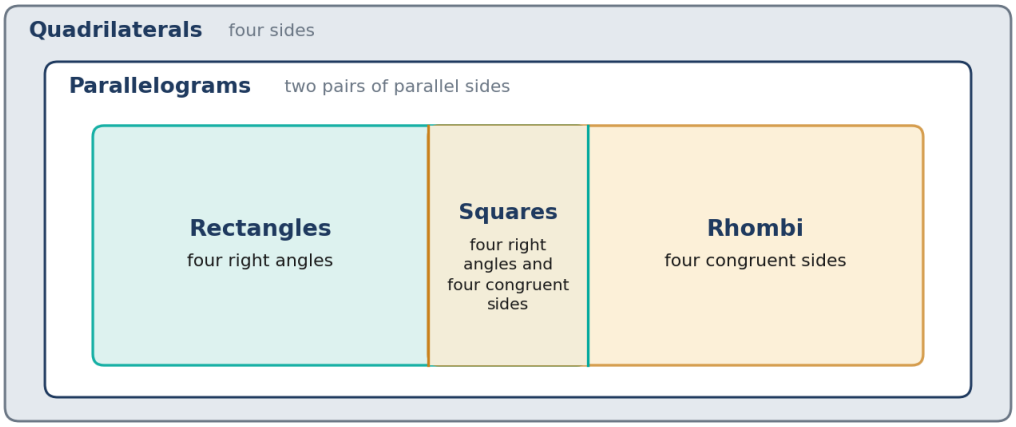



*Figure 3. Squares are both rectangles and rhombi, and all three are parallelograms. DMT Insight 46 examines quadrilateral definitions in detail.*

## Questions That Focus Attention on Properties

Use these questions flexibly. With younger students, begin with the accessible properties. With older students, develop definitions, inclusion relationships, and justification.

- Is the figure closed, and are its sides straight?
- How many sides and vertices does it have?
- What do we know about the side lengths and angles?
- Are any sides parallel or perpendicular?
- Which categories does the figure belong to? How do you know?

Rotation changes a figure's orientation, not its properties.

## Common Student Responses to Investigate

A student's response provides evidence of how the student may be classifying the figure. Ask the student to explain the reasoning before determining whether the response reflects a mistake, an incomplete definition, or a misconception.

- **"That is a diamond."** Ask the student to examine the side lengths and angles. Explain that "diamond" is not a separate mathematical shape. If the figure has four congruent sides and four right angles, it is a square regardless of its orientation. Ask: "What changed when we rotated it? What stayed the same?"

- **“That is not a triangle. It is too skinny.”** The student may be comparing the figure with a familiar example. Ask how many sides and vertices it has.
- **“A square is not a rectangle.”** The student may be treating the categories as exclusive. Ask which properties a rectangle must have.
- **“That is not a real pentagon.”** The student may expect regular shapes. Ask what makes a figure a pentagon.

Varying examples and asking students to examine defining properties can help them classify shapes without relying on orientation or familiar appearance.

## Ways to Examine Geometric Properties

- **Use correct mathematical names:** when students use everyday descriptions such as “diamond,” connect the description to the mathematical properties and identify the figure correctly as a square or rhombus when appropriate.
- **Compare examples and nonexamples:** sort figures, including rotated, irregular, and long and thin examples, and justify each decision with properties.
- **Rotate and reflect:** show the same figure in several orientations and ask whether its name changes, and why.
- **Classify and justify:** place figures in nested categories and explain why a square belongs in more than one.
- **Compose and decompose:** build and take apart shapes with straws, geoboards, or drawn segments, and describe how the sides and angles change.

## Continue With DMT Insight 41

Read DMT Insight 41 for the research behind this Companion. Find it at mathsuccess.io/dmtinsights.