

DMT COMPANION

Insight 37 · 2026 · for teachers, coaches, and school leaders



DEVELOPING TEACHERS' MATHEMATICAL AND INSTRUCTIONAL KNOWLEDGE

Examining teacher knowledge, professional learning, and investment decisions

How to Use This

Teacher knowledge influences the instructional decisions teachers make every day. Understanding the mathematics, recognizing how students develop mathematical ideas, and selecting appropriate tasks and representations are closely connected.

This Companion accompanies DMT Insight 37, which reviews research on mathematical knowledge for teaching, how students learn mathematics, teacher preparation, professional development, and the balance between investing in curriculum and investing in teachers. It offers questions and an example for teachers, coaches, and school leaders as they plan professional learning.

DMTI's approach brings these areas together through sustained professional development, classroom applications, and opportunities to examine student thinking.

Key Idea • Teachers need more than knowledge of procedures. They need to understand the mathematics, how students develop mathematical ideas, and how to design instruction that connects students' thinking to mathematical structure.

Three Guiding Questions

Use these questions for discussion with teachers, coaches, and administrators.

1. What mathematical ideas and relationships do teachers need to understand within the content they teach?
2. How do teachers recognize and interpret students' strategies, mistakes, and misconceptions?
3. How do teachers use models, tasks, and discussions to connect student thinking to mathematical structure?

Three Areas of Teacher Knowledge

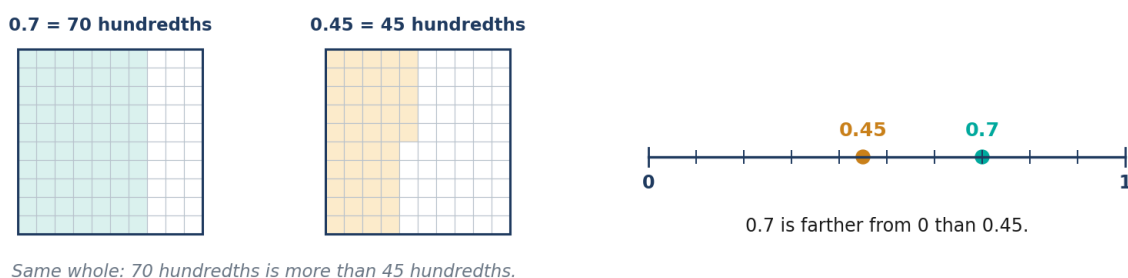
Research distinguishes the mathematics an adult uses to solve a problem from the specialized knowledge teachers use to interpret strategies, choose representations, and explain why procedures work (Ball et al., 2008). Researchers have linked teachers' mathematical knowledge for teaching to students' achievement gains (Hill et al., 2005). The three areas are connected:

- **Mathematical knowledge for teaching:** understanding units, quantities, relationships, operations, equivalence, and mathematical structure, and why procedures work.
- **Knowledge of how students learn mathematics:** understanding learning trajectories, student strategies, mistakes, misconceptions, and how mathematical ideas develop.
- **Pedagogical content knowledge:** selecting tasks and models, interpreting student thinking, facilitating mathematical discussions, and connecting representations to procedures.

Teachers need to understand the mathematics behind procedures and help students develop conceptual understanding and procedural fluency together.

An Example: Comparing 0.45 and 0.7

A student says that 0.45 is greater than 0.7 because 45 is greater than 7. Each area of teacher knowledge shapes the response.



A student says 0.45 is greater than 0.7 because 45 is greater than 7.

What is the unit in each decimal?

Figure 1. Seven tenths is seventy hundredths, which is greater than forty-five hundredths.

- **Mathematical knowledge:** 0.7 is seven tenths, or seventy hundredths, and 0.45 is forty-five hundredths. Comparing the two requires attending to the unit.
- **Knowledge of how students learn:** students often extend whole-number comparison to decimals. The response suggests reasoning, but it is evidence to investigate, not a diagnosis.
- **Pedagogical content knowledge:** ask students to name the unit, represent both decimals on a hundredths grid or number line, and explain why seven tenths is greater than forty-five hundredths.

The questions to ask:

“What is the unit in each decimal?”

“How many hundredths are in seven tenths?”

“Where would each number be on a number line from 0 to 1?”

Developing Teacher Knowledge Through Professional Learning

DMTI’s approach develops these three areas through sustained professional learning, lesson and unit study, classroom applications, and examination of student thinking.

Sustained, Content-Focused Professional Learning

Research on professional development identifies content focus, active learning, coherence, duration, and collective participation as important features (Desimone, 2009; Garet et al., 2001). Professional learning continues over time so that teachers can investigate mathematics, try instructional approaches, and return to what they observe:

- **Summer institutes:** a three-year sequence that studies the mathematics of K–8 content, including units, operations, and equivalence.
- **Classroom coaching and modeling:** DMTI specialists co-teach and model lessons and give feedback on instructional decisions and student reasoning.
- **Lesson and unit study:** teacher teams plan instruction, anticipate student thinking, and analyze student work.

Teachers use what they are learning, examine evidence from their classrooms, and refine their instruction over time.

Connecting Professional Learning to Curriculum

Professional learning should help teachers use their adopted curriculum with attention to the mathematics. Teachers learn to:

- Identify the mathematical ideas and relationships in each unit.
- Select models, such as number lines, arrays, and bar models, that make those relationships visible.
- Facilitate discussions in which students explain and compare their reasoning.
- Supplement where students need more opportunities with a mathematical idea.

The same work applies whether a district uses Eureka Math, Bridges in Mathematics, enVision, or another program.

Using Student Work and Instructional Resources

DMTI provides instructional activities and assessments that help teachers apply what they are learning. Student work from these tasks gives teacher teams evidence to examine: what students understand about the unit, which strategies they use, and what mathematics needs further attention.

Examining the Balance Between Curriculum and Professional Learning

Curriculum materials organize instruction. Teachers' knowledge shapes how they use those materials. In a study of about 6,000 schools in six states, schools using different fourth- and fifth-grade mathematics textbooks showed little difference in average achievement growth, and teachers reported 0.8 to 1.6 days of curriculum-specific training on average (Blazar et al., 2019). DMT Insight 39 examines district spending in more detail.

For the same years, district leaders can document:

- Curriculum purchases, licenses, and replacement materials.
- Curriculum-specific training and publisher sessions.
- Sustained mathematics professional learning, including hours across the year and across years.
- Coaching, lesson and unit study, teacher time, and substitutes.

Examining Evidence of Teacher and Student Learning

Evaluate professional learning by what teachers learn, how instruction changes, and what students come to understand. Teacher reports and participation hours are useful, but they do not, on their own, show that instruction has changed. District leaders can examine evidence in three areas:

- **Teacher knowledge:** growth in mathematical knowledge for teaching, measured with validated assessments where possible.
- **Instruction:** the tasks, models, and questions teachers use, and how students explain their reasoning.
- **Student learning:** state assessments and measures of conceptual understanding and fluency, interpreted alongside how fully the professional learning was implemented.

Questions for School and District Leaders

Use these questions when planning professional learning:

- What mathematics do teachers need to study in the units they teach?
- How will teachers examine student thinking from their own classrooms?
- How will professional learning continue across the year, and across years?
- How is professional learning connected to the curriculum teachers use?
- How will we know what teachers are learning and how instruction is changing?

- How do our investments in curriculum compare with our investments in teachers' mathematical knowledge over the same years?

Continue With DMT Insight 37

Read DMT Insight 37 for the research behind this Companion. Find it at mathsuccess.io/dmtinsights.