

DMT COMPANION

Insight 32 · 2026 · for teachers, coaches, and school leaders



UNDERSTANDING MISTAKES AND MISCONCEPTIONS: WHEN MATHEMATICAL IDEAS NO LONGER WORK

Examining the mathematical ideas behind errors and planning instruction

How to Use This

This Companion is for teachers, coaches, and school leaders working with DMT Insight 32. It gives five examples of a familiar idea applied where it no longer works, with questions to ask, representations that make the difference visible, and ways to respond during instruction. The student responses are illustrative examples, not work from a particular classroom.

Key Idea • An incorrect answer may reflect a mistake in calculation or an idea that no longer applies. Investigate the reasoning before deciding what to teach.

Three Guiding Questions

Consider these questions when a student gives an incorrect answer.

1. What mathematical idea is the student using?
2. Where does that idea work, and where does it no longer apply?
3. What representation, explanation, or practice will develop the relationship that applies?

Familiar Reasoning and Where It Breaks Down

Each line of reasoning below may seem to work in familiar situations, but it is not a valid mathematical rule. Instruction connects the new situation to units, quantities, and relationships.

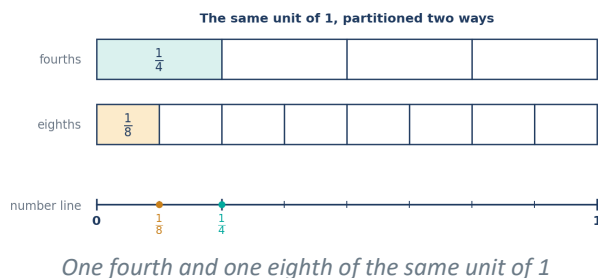
Familiar reasoning	Why it may seem to work	Where it breaks down	Relationship to develop
A larger denominator means a larger fraction	Students compare the whole-number values of denominators	$\frac{1}{8} < \frac{1}{4}$, using the same unit of 1	More equal partitions of the same unit of 1 produce smaller unit fractions.
The equal sign means “the answer comes next”	Many early equations are written as $3 + 5 = 8$	$8 = 3 + ?$	Both sides represent equivalent quantities

Familiar reasoning	Why it may seem to work	Where it breaks down	Relationship to develop
Subtract the smaller digit from the larger.	The shortcut gives a correct answer when no regrouping is needed, as in $58 - 16$	$52 - 16$	Decompose one ten into ten ones
Longer numerals represent larger numbers	Often appears true when comparing positive whole numbers without leading zeros	$0.45 < 0.7$	Compare the values of place-value units
Division makes the number smaller	Common early examples divide positive quantities by whole numbers greater than 1	$12 \div \frac{1}{2} = 24$	Division can determine how many units of the divisor are contained in the dividend.

Worked Examples

Example 1 • fractions and the unit of 1

A student says that $\frac{1}{8}$ is greater than $\frac{1}{4}$ because 8 is greater than 4. Show both fractions with the same unit of 1. Partitioning the unit of 1 into more equal parts produces a smaller fractional unit.



Questions to ask:

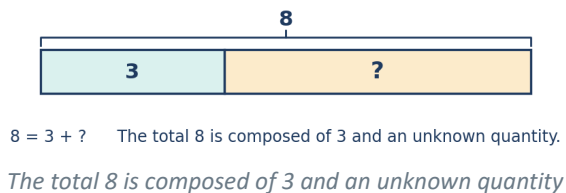
"What is the unit of 1? Show it."

"Into how many equal parts did you partition the unit of 1? How does that change the size of each part?"

"Where are $\frac{1}{8}$ and $\frac{1}{4}$ on the number line? Which is farther from 0?"

Example 2 • the equal sign as a relationship

A student says that $8 = 3 + ?$ is backward. Represent 8 as the total composed of 3 and an unknown quantity. Then discuss equations such as $8 = 8$ and $2 + 6 = 5 + 3$ as statements that both sides represent the same quantity.



Questions to ask:

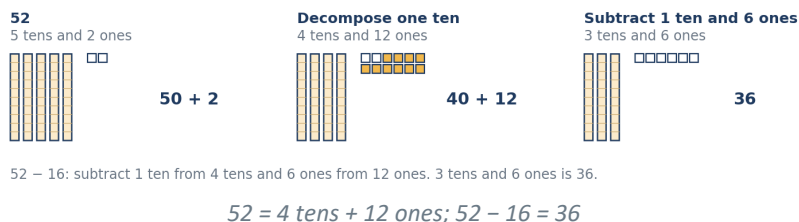
“What does the equal sign tell us about the two sides?”

“Which part of the bar model is 8? Which part is unknown?”

“Is $2 + 6 = 5 + 3$ true? How do you know?”

Example 3 • place value and subtraction

A student calculates $52 - 16 = 44$ by subtracting the smaller digit from the larger in each column. The difference is 36. Represent 52 as 5 tens and 2 ones, decompose one ten into ten ones, and connect each step to the written procedure.



Questions to ask:

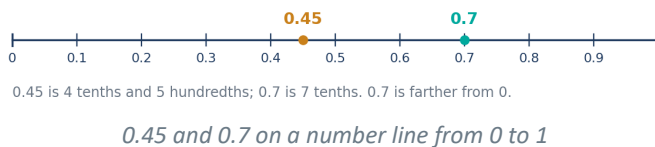
“What do the 5 and the 2 represent?”

“How can we represent 52 so that we can subtract 6 ones?”

“Where does the written procedure show the ten you decomposed?”

Example 4 • comparing decimals

A student says that 0.45 is greater than 0.7 because 45 is greater than 7. Compare the place-value units: 0.45 is 4 tenths and 5 hundredths, and 0.7 is 7 tenths, or 70 hundredths. On a number line, 0.7 is farther from 0.



Questions to ask:

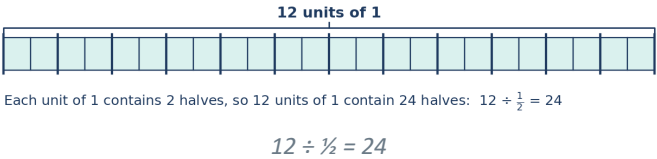
“What does the 4 in 0.45 represent? What does the 7 in 0.7 represent?”

“How many hundredths are in 0.7?”

“What quantity does each decimal represent? How do you know which is greater?”

Example 5 · dividing by a unit fraction

A student says that $12 \div \frac{1}{2} = 6$, perhaps because division has made numbers smaller in earlier work. The division asks how many halves are in 12. Each unit of 1 contains 2 halves, so 12 units of 1 contain 24 halves.



Questions to ask:

- “What does $12 \div \frac{1}{2}$ ask us to determine?”
- “How many halves are in one unit of 1? In 12 units of 1?”
- “When does dividing give a smaller number, and when does it not?”

Responding During Instruction

Not every error needs an extended discussion. Match the response to the evidence.

If the evidence suggests	A possible response
A calculation or recording slip, and the student can explain the correct reasoning.	Specific feedback, then similar practice
The reasoning is unclear.	Ask the student to explain or represent the quantities before deciding.
An idea applied where it no longer works.	Show where the idea works, where it no longer applies, and teach the relationship that does
Several students share the same idea.	Discuss an anonymous example, keeping the focus on the mathematics.
The relationship is understood but not yet efficient	Varied practice to develop accuracy, efficiency, and flexibility

Questions That Help Investigate Student Reasoning

Direct correction is often appropriate: “That answer is incorrect; let’s examine why.” The questions below help you investigate reasoning once you identify an error.

Error	A question to ask
Any incorrect answer	“That answer is incorrect. How did you decide? Show me with a model.”
$\frac{1}{8}$ is greater than $\frac{1}{4}$	“What is the unit of 1? How many equal parts did you make?”
$8 = 3 + ?$ is “backward”	“What does the equal sign tell us about the two sides?”
$52 - 16 = 44$	“Decompose one ten into ten ones. How many ones do you have now?”

Across the Grades

The content changes across grade levels. The questions about where an idea works and where it no longer applies stay the same.

Grade band	Familiar reasoning	Where it no longer applies	Relationship to develop
Grades K–2	The equal sign means “the answer comes next”	$8 = 3 + ?$	Equivalence of both sides
Grades 3–5	A larger denominator means a larger fraction; longer numerals are larger	$\frac{1}{2}$ and $\frac{1}{4}$; 0.45 and 0.7	The unit of 1 and place-value units
Grades 6–8	Greater absolute value means a greater number	-7 and -3	Compare signed numbers by their position relative to 0 on a number line

For Coaches

Coaches can work with teachers to collect common incorrect responses, identify the idea behind each one, and plan tasks that show where the idea works and where it no longer applies.

Questions for a coaching conversation:

- Which incorrect answers appeared most often, and what idea might each reflect?
- Which errors needed only feedback and practice?
- What representation made the difference visible?
- How did we connect the representation to the written procedure?

For School Leaders

School leaders can support this work by giving teachers time to examine student work together and by treating incorrect answers as evidence for instruction rather than as a reflection of the student.

Where to Go Next

Read DMT Insight 32 at mathsuccess.io/dmtinsights.

Choose one error your students make often. Identify the idea behind it, where that idea works, and a task that shows where it no longer applies.