

DMT COMPANION

Insight 31 · 2026 · for teachers, coaches, and school leaders



HOW MATHEMATICAL UNDERSTANDING DEVELOPS AND LASTS

Planning instruction and practice so students retain and apply mathematics

How to Use This

This Companion is for teachers, coaches, and school leaders working with DMT Insight 31. It turns the research on prior knowledge, working memory, and practice into planning decisions, examples, and questions. Start with the three questions that follow.

Key Idea • Mathematical understanding and procedural fluency develop through instruction, reasoning, feedback, and practice. Retrieval, spacing, and interleaving can help students retain and apply what they learn.

Three Guiding Questions

Answer these before you read the Insight, then again afterward.

1. What prior knowledge will students bring to this mathematics, and how might it help or interfere?
2. How often do students return to important relationships after the unit in which they learned them?
3. When students practice, what opportunities do they have to select and explain an appropriate strategy?

From Research to Planning

Each research finding suggests a planning decision.

Research idea	What the research shows	Planning decision
Prior knowledge	Students interpret new ideas through what they already know (Ausubel, 1968; Piaget, 1965)	Find out what students understand about the relationship before the lesson.
Working memory	Capacity for unfamiliar information is limited (Cowan, 2001)	Choose examples and representations that focus attention on the relationship
Retrieval practice	Recalling information strengthens later retention (Roediger & Karpicke, 2006)	Provide opportunities for students to recall and use previously learned mathematics without immediately supplying the procedure
Spaced practice	Practice spread over time improves long-term retention (Cepeda et al., 2006)	Return to important ideas across weeks and units.
Interleaved practice	Mixing problem types improves delayed performance (Rohrer et al., 2015)	After appropriate initial instruction and practice, mix related problem types so students determine which strategy applies

Worked Examples

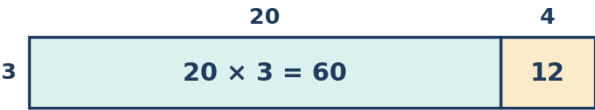
Example 1 · prior knowledge: 24×3

A response

$24 \times 3 = 612$

$2 \times 3 = 6$ and $4 \times 3 = 12$,
written side by side

The area model



$60 + 12 = 72$

The 2 in 24 is 2 tens, so 2×3 is 6 tens, or 60.

Decomposing 24 into 20 and 4 shows how the distributive property connects place value and multiplication.

The student may know the multiplication facts but may not be accounting for the value of each digit. Ask the student to explain the calculation before deciding how to respond. Decomposing 24 into 20 and 4 connects the facts to place value and the distributive property: $(20 \times 3) + (4 \times 3) = 60 + 12 = 72$.

Questions to ask:

“How did you get 612? Show me with a model or a drawing.”

“What does the 2 in 24 represent? What is 20×3 ?”

“How are 60 and 12 represented in the model? How should we combine these quantities?”

Example 2 • retrieval and spacing: returning to the relationship

After the lesson, the same relationship returns in short retrieval prompts spread across several weeks. Each prompt asks students to use and explain the relationship, not only produce an answer.

When	Retrieval prompt
Next day	“Find 24×3 . Show how you decomposed 24.”
One week later	“Find 36×4 . What are the partial products?”
Three weeks later	“A student wrote $52 \times 3 = 156$. Show how place value and the distributive property can justify the answer.”
Six weeks later	“Find 4×27 two different ways.”

Example 3 • interleaving: deciding which relationship applies

After students practice each strategy, a mixed set asks them to decide which relationship fits the numbers. For example: 25×4 , 19×5 , 24×3 , and 50×6 . A student might use $25 \times 4 = 100$ directly, find 19×5 as $20 \times 5 - 5$, decompose 24 into 20 and 4, and use $5 \times 6 = 30$ for 50×6 . This mixed practice gives students opportunities to select and compare strategies based on the numbers and mathematical relationships involved.

Questions to ask:

“Which problem would you solve first? Why?”

“Which relationship helped with 19×5 ?”

“Which strategy was most efficient for each problem?”

Key Idea • Students develop fluency by practicing calculations, examining mathematical relationships, and determining which strategies are accurate and efficient.

Questions That Reveal Student Reasoning

Both columns have a place. The right column adds questions that show what students retain and why.

Instead of only	Also ask
“Remember the steps from yesterday.”	“What did we figure out yesterday about 24? How can you use it today?”
“Do these twenty problems of the same kind.”	“Which of these problems can you solve the same way? Which need a different strategy?”
“What is 6×4 ?”	“How could you find 6×4 if you forgot it?”
“Is your answer right?”	“How can you check that your answer makes sense?”

For Coaches

Coaches can work with teachers to examine how important mathematical relationships develop across lessons and units. Reviewing student work provides evidence of what students understand, what they can retrieve, and whether they can apply their knowledge to different problems. This evidence can guide decisions about additional instruction, retrieval opportunities, and mixed practice.

Questions for a coaching conversation:

- What evidence shows that students understand and can use mathematical relationships developed in earlier units?
- Where in the week do students recall earlier mathematics without being reminded?
- When does practice ask students to decide which strategy applies?
- What does student work show about errors that return after a few weeks?

For School Leaders

Pacing guides and assessments shape whether students revisit important ideas. Leaders can build cumulative review into pacing, include earlier content on unit assessments, and protect planning time for teachers to design retrieval and mixed practice.

Across the Grades

The same research applies across grade levels. The mathematics changes.

Grade band	A relationship to develop	Retrieval prompt	Mixed practice
Grades K–2	Composing ten: $8 + 5$ as $8 + 2 + 3$	“How can you use 10 to find $7 + 6$?”	Mix make-ten, doubles, and counting on
Grades 3–5	Decomposing a factor: 24×3 as $(20 \times 3) + (4 \times 3)$	“Find 36×4 . What are the partial products?”	Mix decomposing, compensating, and known facts
Grades 6–8	The distributive property: $3(x + 4) = 3x + 12$	“Write $5(x + 2)$ another way. How do you know?”	Mix equivalent expressions requiring the distributive property, factoring, or combining like terms

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Read DMT Insight 31 for the research behind this Companion. Find it at mathsuccess.io/dmtinsights.