

DMT COMPANION

Insight 30 · 2026 · for teachers, coaches, and school leaders



PROGRESSIVE FORMALIZATION: FROM MATHEMATICAL RELATIONSHIPS TO FORMAL PROCEDURES

Connecting units, representations, and the standard algorithm in multiplication

How to Use This

This Companion is for teachers, coaches, and school leaders working with DMT Insight 30. It provides worked examples that connect quantities, representations, and the standard multiplication algorithm; questions to ask; and ways to use student responses as evidence for planning instruction.

Key Idea • Students connect units, quantities, and relationships to formal notation and procedures. Conceptual understanding and procedural fluency develop together.

Three Guiding Questions

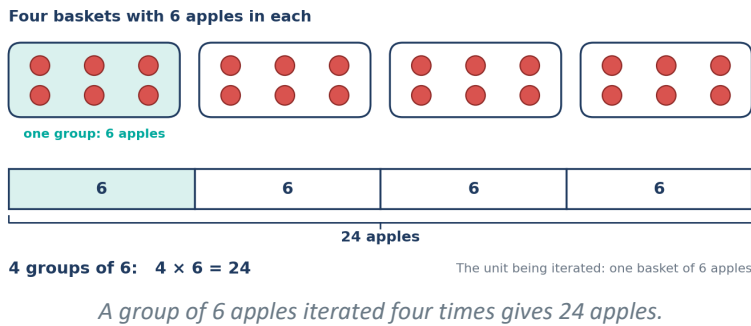
Consider these questions as you plan a lesson and examine student work.

1. What unit is being iterated, and what does each number represent?
2. Which representation will help students examine the quantities and relationships involved?
3. Can students explain how the formal procedure represents the same relationship?

Worked Examples

Example 1 • identify the unit: 4 baskets of 6 apples

Four baskets contain 6 apples each. Students identify 6 apples as one group and iterate that group four times. The four equal groups compose 24 apples, represented by $4 \times 6 = 24$. The bar model shows the same relationship.

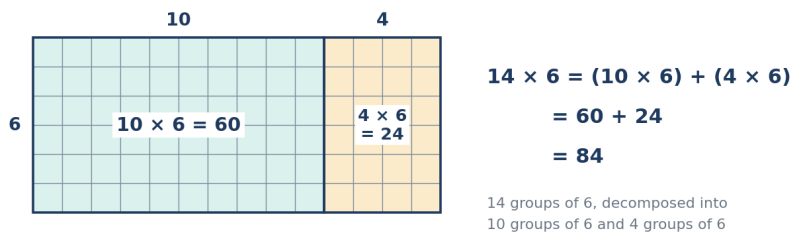


Questions to ask:

- “What is one group? How many groups are there?”
- “What does the 4 represent? What does the 6 represent?”
- “Where is the 24 in the bar model?”

Example 2 · decompose: 14×6 with an area model and a ratio table

Students decompose 14 into 10 and 4, giving 10 groups of 6 and 4 groups of 6. The area model and the ratio table show the same partial products, and the distributive property explains why they can be added: $14 \times 6 = (10 \times 6) + (4 \times 6) = 60 + 24 = 84$.



14 groups of 6, decomposed into 10 groups of 6 and 4 groups of 6: $14 \times 6 = (10 \times 6) + (4 \times 6) = 60 + 24 = 84$

Number of groups	1	2	4	10	14
Number of apples	6	12	24	60	84

Questions to ask:

- “Why decompose 14 into 10 and 4?”
- “Where are 10×6 and 4×6 in the area model? In the ratio table?”
- “How does the distributive property explain the two partial products?”

Example 3 · connect the partial products to the standard algorithm.

The standard algorithm organizes the calculation using place value. Multiplying 6 by 4 ones gives 24 ones, recorded as 4 ones and 2 regrouped tens. Multiplying 6 by 1 ten gives 6 tens. Combining these with the 2 regrouped tens gives 8 tens, or 84. Teachers can explicitly connect these steps to the partial products of 24 and 60 and provide practice to develop accuracy and efficiency.

Partial products

$$\begin{array}{r}
 14 \\
 \times 6 \\
 \hline
 24 \\
 + 60 \\
 \hline
 84
 \end{array}$$

4 ones $\times$ 6 = 24 ones
1 ten $\times$ 6 = 6 tens, or 60

Standard algorithm

$$\begin{array}{r}
 2 \quad 24 \text{ ones} = 2 \text{ tens and 4 ones:} \\
 14 \quad \text{write 4, regroup the 2 tens} \\
 \times 6 \\
 \hline
 84 \quad 6 \text{ tens} + 2 \text{ tens} = 8 \text{ tens}
 \end{array}$$

The 24 ones are recorded as 4 ones and 2 regrouped tens.
The 6 tens from 6×10 (the 60) and the 2 regrouped tens make 8 tens.

Connect the digits and regrouping to the quantities represented by the partial products. The 60 appears as the 6 tens from 6×10 .

Questions to ask:

“Where is the 24 in the algorithm? Where is the 60?”

“What does the regrouped 2 represent?”

“Why is the 8 in the tens place?”

Key Idea • Students do not need every representation before the algorithm. Select the representation that makes the unit, the decomposition, and place value visible.

What Student Responses Can Show

A response is evidence to examine. Ask the student to explain before deciding what instruction should follow.

Response to 14×6	What to investigate	Possible teacher response
Writes 64	Whether the student accounted for the 2 regrouped tens	Ask the student to explain the regrouping and compare with $24 + 60$
Writes 24	Whether the student calculated only 6×4 and omitted the 6 tens from 6×10	Ask the student to show each step and explain what the 1 in 14 represents.
Finds 84 with partial products but not the standard algorithm	How the student understands the place-value organization of the algorithm	Connect the partial products to the digits and regrouping
Finds 84 with the algorithm but cannot explain the regrouped 2	How the student interprets the regrouped quantity	Represent 24 ones as 2 tens and 4 ones

Questions That Extend Mathematical Reasoning

Questions about answers and procedures are important. Teachers can also ask students to explain the units, quantities, and relationships their calculations represent.

Instead of only	Also ask
"What is 14×6 ?"	"How could you use 10×6 to find 14×6 ?"
"Regroup the 2 tens."	"What does the 2 represent? Where does it come from?"
"Draw an array."	"How does your array show 10 groups of 6 and 4 groups of 6?"
"Is that right?"	"How do the partial products show that 84 makes sense?"

Across the Grades

The same approach applies across grade levels—the mathematics changes.

Grade band	Relationship	Representations	Formal procedure
Grades K–2	$8 + 5$ composed through 10: $8 + 2 + 3$	Ten-frames, number line	Addition equations and efficient strategies based on composing ten
Grades 3–5	$14 \times 6 = (10 \times 6) + (4 \times 6)$	Area model, ratio table	The standard multiplication algorithm
Grades 6–8	$2(x + 3) = 2x + 6$	Area model with sides 2 and $x + 3$	Applying the distributive property to expand expressions

For Coaches

Coaches can work with teachers to identify the mathematical relationships in a lesson, select representations that support those relationships, and examine student responses to plan subsequent instruction.

Questions for a coaching conversation:

- Which units and quantities did students identify, and how did they represent them?
- Where did students connect the partial products to the standard algorithm?
- Where did instruction, feedback, and practice build accuracy and efficiency?
- What do the student responses suggest for the next lesson?

For School Leaders

School leaders can support this work through curriculum that connects representations and procedures, collaborative planning time, and assessments that examine conceptual understanding and procedural fluency. Read DMT Insight 30 at mathsuccess.io/dmtinsights.