

DMT COMPANION

Insight 25 · 2026 · for teachers, coaches, principals, and families



BEYOND BORROWING: UNDERSTANDING THE STRUCTURE OF SUBTRACTION

Composing and decomposing place-value units in subtraction

Examining Regrouping Through Place-Value Units

This Companion is for anyone working with DMT Insight 25. The Insight reviews the research on place value, regrouping, and the strategies students develop for multi-digit subtraction. Use the models and questions in this Companion to examine how students reason about regrouping. Start with the three questions that follow.

Key Idea • Regrouping decomposes a unit into ten units of the next smaller place. The quantity does not change.

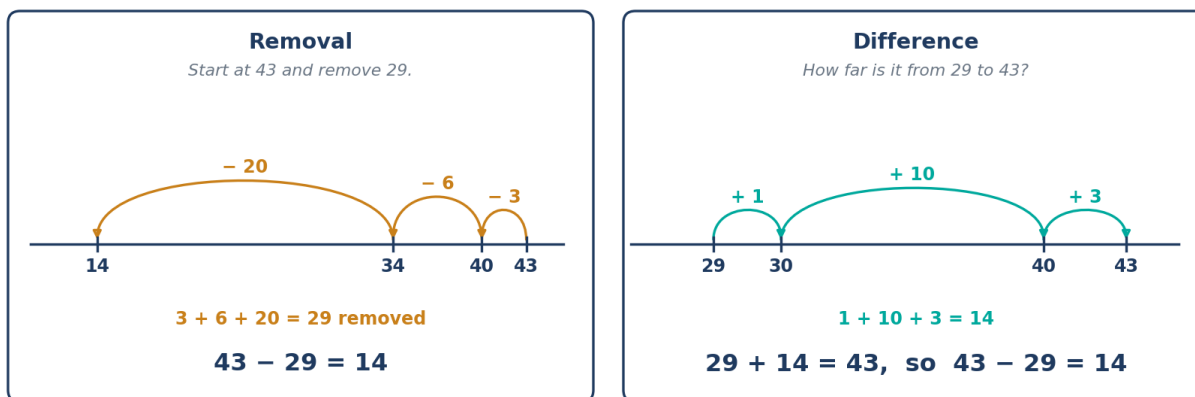
Three Guiding Questions

Use these questions for discussion with teachers and coaches.

1. When your students regroup, can they name the unit they decomposed?
2. Can they explain why 3 tens and 13 ones is still 43?
3. What do they do when there are no tens to decompose, as in $402 - 178$?

One Subtraction, Several Strategies

For $43 - 29$, students might remove 29 in jumps or count up from 29 to 43. DMT Insight 45 examines removal and difference in detail. This Companion focuses on how students compose and decompose place-value units.



The difference is the same. The relationship each strategy uses is different.

Ask students to explain how each strategy uses tens and ones. The standard regrouping algorithm records the decomposition of place-value units.

Key Idea • Different strategies use place-value relationships in different ways. Ask students to explain the units and quantities involved.

Worked Examples: The Model and the Questions to Ask

Ask the same question at each place, starting with the ones:

- **Do we have enough ones to subtract?** If not, decompose 1 ten into 10 ones.
- **Do we have enough tens to subtract?** If not, decompose 1 hundred into 10 tens.
- **Are there no tens to decompose?** Decompose 1 hundred into 10 tens first, then 1 ten into 10 ones.
- **Did the total quantity change?** It should not. Only the representation changed.

Example 1 • decompose a ten: $43 - 29$

43



↓ decompose 1 ten into 10 ones



$30 + 13 = 43$. The quantity is equal.

The written procedure records the decomposition

$$\begin{array}{r} 3 \quad 13 \\ \cancel{4} \quad \cancel{3} \\ - 2 \quad 9 \\ \hline 1 \quad 4 \end{array}$$

13 ones – 9 ones = 4 ones

3 tens – 2 tens = 1 ten

What unit was decomposed? Did the total quantity change?

In $43 - 29$, one ten is decomposed into ten ones. The representation changes from 4 tens and 3 ones to 3 tens and 13 ones. The quantity stays 43.

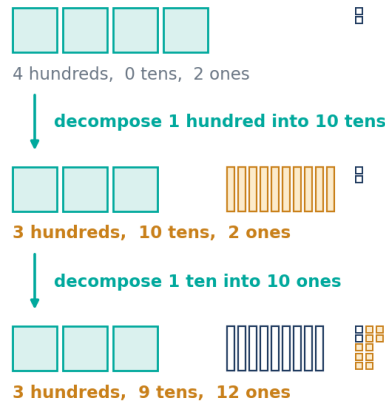
The questions to ask:

“There are 3 ones. Can we subtract 9 ones? What unit could we decompose?”

“How many tens do we have now? How many ones?”

“Is it still 43? How do you know?”

Example 2 • decompose across a zero: 402 – 178



The written procedure records both decompositions

3	9	12	
4	0	2	12 ones – 8 ones = 4 ones
– 1	7	8	9 tens – 7 tens = 2 tens
2	2	4	3 hundreds – 1 hundred = 2 hundreds

There are no tens in 402, so a hundred is decomposed first.

What unit was decomposed? Did the total quantity change?

Each representation is 402. The quantity is equal.

In 402 – 178, there are no tens to decompose. One hundred is decomposed into ten tens, and then one ten into ten ones. 3 hundreds, 9 tens, and 12 ones is still 402.

The questions to ask:

“There are no tens. What larger unit can we decompose first?”

“What do the 3, the 9, and the 12 above 402 represent?”

Key Idea • A model earns its place when it shows what happens to the units.

What Student Errors Can Reveal

A response is evidence of how a student is reasoning, not a diagnosis. The interpretations below are possibilities to investigate with the student.

$\begin{array}{r} 4\ 3 \\ - 2\ 9 \\ \hline 2\ 6 \end{array}$	ones	“Nine minus three is six.”	9 – 3, not 3 – 9
	tens	“Four minus two is two.”	4 tens – 2 tens

a student’s response

The correct difference is 14.

The student subtracts the smaller digit from the larger digit in each column. An estimate, 43 – 30 = 13, also shows that 26 is too large.

What You See	Possible Interpretation	Ask
43 – 29 = 26	The student may be subtracting the smaller digit from the larger digit in each column: 9 – 3 = 6 and 4 – 2 = 2.	“Which number are you subtracting from? How many ones are in 43?”
43 – 29 = 24	The student may have decomposed a ten to make 13 ones (13 – 9 = 4) but subtracted the tens from 4 tens instead of 3 tens (4 – 2 = 2).	“How many tens are left after you decompose one ten?”

$402 - 178 = 376$	The student may be subtracting the smaller digit from the larger digit in each place: $8 - 2 = 6$, $7 - 0 = 7$, and $4 - 1 = 3$.	"How many tens are in 402? What could we decompose to get some?"
$402 - 178 = 324$	The student may have written 9 tens and 12 ones but left 4 hundreds: $12 - 8 = 4$, $9 - 7 = 2$, and $4 - 1 = 3$. The hundred that was decomposed is still counted as a hundred.	"How many hundreds are left after you decompose one hundred?"

Key Idea • An incorrect answer can provide evidence of how a student is reasoning. Ask what relationship or procedure produced the answer before correcting it.

Questions That Connect Procedures to Units

Replace the statement on the left with the question on the right. The question connects the written procedure to the units and the quantity.

Instead of saying	Ask
"Borrow from the tens."	"What unit can we decompose? Does the quantity change?"
"You can't take 9 from 3."	"Are there enough ones? What unit could we decompose to get more ones?"
"Cross out the 4 and write 3."	"What does the 3 represent? Where did the other ten go?"
"Borrow from the zero."	"There are no tens. What larger unit can we decompose first?"
"Follow the steps."	"How does the written procedure show what happened to the units?"

What to look for in a classroom:

- Students name the unit they decomposed and say that the total quantity stayed equal.
- Students explain the marks they write above the digits.
- Students decompose a hundred into tens when there are no tens, as in $402 - 178$.
- Students compare strategies, such as counting up and decomposing a ten, and say which units each one uses.

Across the Grades

Subtraction continues across mathematics. The relationships stay the same. The quantities and units change.

Grade band	Example	Model	The shift it asks for
Grades K–1	Composing and decomposing ten within 20, such as $13 - 5$ as $13 - 3 - 2$	Ten-frames, counters	From counting by ones to using ten as a unit
Grades 2–3	Decomposing tens and hundreds, such as $43 - 29$ and $402 - 178$	Base-ten blocks, open number lines	From “borrowing” to naming the unit that was decomposed
Grades 4–5	Multi-digit algorithms and decimal subtraction, such as $3.2 - 1.75$	Place-value charts	From following steps to connecting the algorithm to place-value units
Grades 6–7	Difference and distance with rational numbers, including negative values, such as $3 - (-2)$	Number lines	From removal to distance, including distance across zero

The same relationship with decimals:

In $3.20 - 1.75$, there are no hundredths in 3.20 and not enough tenths. One one is decomposed into ten tenths, and then one tenth is decomposed into ten hundredths:

$$\begin{aligned} 3.20 &= 3 \text{ ones} + 2 \text{ tenths} + 0 \text{ hundredths} \\ &= 2 \text{ ones} + 12 \text{ tenths} + 0 \text{ hundredths} \\ &= 2 \text{ ones} + 11 \text{ tenths} + 10 \text{ hundredths} \end{aligned}$$

Then $3.20 - 1.75 = 1.45$. The units are smaller, but the sequence is the same as in $402 - 178$.

Key Idea • The units change from hundreds and tens to tenths and hundredths. The relationship stays the same: decompose one unit into ten of the next smaller unit.

Continue With DMT Insight 25

Read DMT Insight 25 for the research behind this Companion. Find it at mathsuccess.io/dmtinsights.

For discussion: when your students write 3, 9, and 12 above 402 in $402 - 178$, what do they say those marks represent, and how do they explain why the quantity is still 402?