

DMT COMPANION

Insight 23 · 2025 · for teachers, coaches, and school leaders



ASSESSING MATHEMATICAL UNDERSTANDING: FROM STUDENT RESPONSES TO INSTRUCTION

Examining strategies, representations, and explanations for $36 + 27$

How to Use This

This Companion uses $36 + 27$ to examine the four DMT assessment categories, compare student strategies, and consider how teachers can use the responses to plan instruction. The student responses are illustrative examples, not work collected from a particular classroom.

Key Idea • A student response is evidence to investigate. Ask the student to explain before deciding what to teach next.

Three Guiding Questions

Consider these questions as you examine student work.

1. What strategy did the student use, and which units did the student work with?
2. What mathematical relationship does the representation or explanation show?
3. What evidence do I still need, and what will I teach next?

Four Categories, One Problem

Each task below uses $36 + 27$. Comparing them shows the different evidence each category can provide.

DMT category	Task	What to Examine
Level 1: Skills and Procedures	Calculate $36 + 27$.	Accuracy, efficiency, and the procedure used
Level 2: Problem Solving	A class collected 63 cans over two days. They collected 36 cans on Monday. How many cans did they collect on Tuesday? Show how you determined the missing quantity and explain how the quantities are related.	How the student interprets the missing quantity and explains how 36, 27, and 63 are related

DMT category	Task	What to Examine
Level 2: Conceptual Understanding	Use a bar model or place-value representation to show how 36 and 27 compose 63. Explain how ten ones can be composed into one ten.	Whether the student represents tens and ones and composes a ten
Level 3: Reasoning and Justification	A student says, "I added 30 and 20 to get 50, then 6 and 7 to get 13, so the total is 63." Explain why the strategy works. Would it work for other two-digit numbers, whether or not the ones compose another ten? Justify your answer.	Whether the student explains why the total is preserved and justifies a general claim

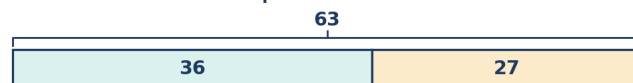
Key Idea • A model, a context, or a request for an explanation does not decide the category by itself. The category depends on the mathematical knowledge and reasoning the task is designed to assess.

Worked Examples

Example 1 • composing 36 and 27, Level 2 Conceptual Understanding

The bar model shows 36 and 27 as parts of the total of 63. In tens and ones, 36 and 27 make 5 tens and 13 ones. Ten of the ones compose one more ten, so the total is 6 tens and 3 ones.

Bar model: 36 and 27 compose 63



36 + 27 in tens and ones: 5 tens and 13 ones



Compose ten ones into one ten: 6 tens and 3 ones



The bar lengths are proportional. The darker ten is composed from ten ones.

$$36 + 27 = 5 \text{ tens} + 13 \text{ ones} = 6 \text{ tens} + 3 \text{ ones} = 63$$

Questions to ask:

“What does each part of the bar represent? What does the bracket represent?”

“How many ones do you have altogether? How many tens can you compose from them?”

“How does your drawing match each step you wrote?”

Example 2 • six responses to $36 + 27$

The responses below are illustrative. Each one is a starting point for a question, not a conclusion about the student.

Response	Possible Interpretation	Question to ask	Possible next step
63, using the written algorithm with the regrouped 1 recorded	An accurate procedure; understanding of the regrouping is not yet visible	“What does the 1 above the tens represent?”	If the student explains one ten, practice for fluency
513	Possibly the ones and the tens added separately, 13 and 5, without composing a ten.	“What do the 5 and the 13 represent? How many tens can you make from 13 ones?”	If the explanation confirms it, work composing ten ones into one ten, connected to the written record.
53	The student may have omitted the composed ten, or made another calculation or recording error.	“Show 36 and 27 with tens and ones. How many tens are there altogether?”	Decide after the student explains: feedback on recording the composed ten, or on the calculation
$36 + 4 = 40$, $40 + 23 = 63$	27 decomposed to compose a ten with 36	“Why did you decompose 27 into 4 and 23?”	Compare with other strategies; try $48 + 35$
$30 + 20 = 50$, $6 + 7 = 13$, $50 + 13 = 63$	Both numbers decomposed into tens and ones	“Why can you decompose and recompose the quantities without changing the total?”	Pose the Level 3 task about other two-digit numbers.
63, counting on by ones from 36	An accurate answer from counting single units; use of units of ten is not yet visible	“How many tens are in 27? Could you solve it another way?”	If the student uses units of ten, compare with a more efficient strategy; if not, work on iterating the unit of ten

Key Idea • A correct answer can come from different strategies. An incorrect answer can come from different sources. Ask before deciding.

Example 3 • examining a general claim, Level 3 Reasoning and Justification

A student says, “I added 30 and 20 to get 50, then 6 and 7 to get 13, so the total is 63.” The strategy works because $36 = 30 + 6$ and $27 = 20 + 7$, and decomposing and recomposing the quantities does not change the

total. Students can test whether the same way of composing tens and ones works for other two-digit numbers, such as $48 + 35$.

Questions to ask:

“Why can you add the tens and the ones separately and still get the total?”

“What happens when the ones make more than ten? Show it with $48 + 35$.”

“Will this work for any two two-digit numbers? How do you know?”

From Evidence to Instruction

Match the next step to what the evidence shows. One response rarely settles the question, so gather more evidence when it is unclear.

If the evidence suggests	A possible next step
A relationship needs further development, such as composing ten ones into one ten.	Tasks that ask students to represent, compose, and decompose the units, with discussion of what stays the same
The procedure is not connected to the quantities.	Explicit instruction linking each step of the written procedure to a representation
An error in calculation or recording	Specific feedback on the error, then a similar task
The understanding is in place but not yet efficient	Varied practice to develop fluency

A Planning Cycle

Teachers and teams can use these four steps:

- Select a mathematical relationship students are learning.
- Design tasks that examine different aspects of that knowledge.
- Examine student strategies, representations, and explanations.
- Use the evidence to plan instruction, feedback, and practice.

Across the Grades

The content changes across grade levels. The categories and the questions about units and relationships stay the same.

Grade band	Level 1 task	Level 2 Conceptual task	Level 3 task
Grades K–2	$8 + 5$	Show how 8 and 5 compose 13 as 10 and 3	Is $8 + 5$ the same as $10 + 3$? Justify
Grades 3–5	$3/4 + 1/8$	Partition the unit of 1 to show $3/4$ as $6/8$, then add $1/8$	A student says $3/4 + 1/8 = 4/12$. Evaluate the claim
Grades 6–8	Solve $3x + 4 = 19$	Use a bar model to show three iterations of the unknown quantity, together with 4, composing a total of 19. Explain how the model represents $3x + 4 = 19$.	Is $3(x + 4)$ the same as $3x + 4$? Justify

For Coaches

Coaches can work with teachers to design tasks across the categories for the same mathematical relationship, examine a set of student responses together, and use the responses as evidence for planning rather than as diagnoses.

Questions for a coaching conversation:

- Which categories do our classroom tasks examine, and which are missing?
- Which responses were correct but not yet explained?
- What questions did we ask before deciding what a response meant?
- What will the next lesson develop, and how will we know?

For School Leaders

School leaders can support this work by giving teachers time to examine student work together, encouraging common tasks across classrooms, and treating student responses as instructional evidence rather than scores alone.

Where to Go Next

Read DMT Insight 23 at mathsuccess.io/dmtinsights.

Select one problem from an upcoming lesson. Write a task for each category, collect the responses, and sort them by strategy before deciding what to teach next.