

DMT COMPANION

Insight 21 · 2026 · for teachers, coaches, and school leaders



DEVELOPING MULTIPLICATIVE REASONING: UNITS, ITERATION, AND STRUCTURE

Connecting bar models, area models, and multiplication procedures

How to Use This

This Companion provides worked examples and instructional questions for teachers, coaches, and school leaders. The examples show how units, iteration, and decomposition connect bar models, area models, partial products, and standard multiplication procedures. Student responses provide additional evidence for planning instruction.

Key Idea • Multiplication involves a unit, the number of times it is iterated, and the resulting total. Conceptual understanding and procedural fluency develop together.

Three Guiding Questions

Consider these questions as you plan a lesson and examine student work.

1. What is the unit, how many times is it iterated, and what is the total?
2. How can the unit or the number of iterations be decomposed, and how do the parts compose the total?
3. How does each representation show the same relationship as the equation and the procedure?

Structural Language

Use these terms with the representations and procedures students are working with.

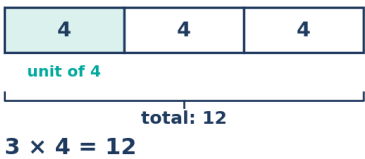
Term	Meaning	In 3×4
Unit	A quantity treated as one unit that can be iterated or composed with other units	4
Iterate	Repeat a unit a specified number of times	The unit of 4 is iterated 3 times
Compose	Combine quantities or units to form another quantity	Three iterations of the unit of 4 compose 12
Decompose	Partition a quantity into parts while preserving its total value	$3 \times 4 = (2 \times 4) + (1 \times 4)$
Total	The resulting quantity represented by the expression	12

Worked Examples

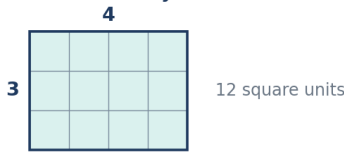
Example 1 · units and iteration: 3×4

Three iterations of the unit of 4 produce a total of 12. The bar model shows the unit being iterated. The area model represents the same product through a rectangle with dimensions 3 and 4.

Bar model: 3 iterations of the unit of 4



Area model: 3 by 4



Three iterations of the unit of 4 in a bar model and an area model.

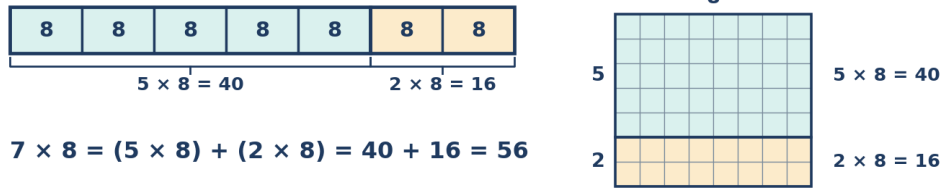
Questions to ask:

- “What is the unit? How many times is it iterated?”
- “Where is the total in the bar model? In the area model?”
- “How could we decompose the three iterations?”
- “How does the area model show the relationship between the two dimensions and the total area?”

Example 2 · decomposing iterations: 7×8

Seven iterations of the unit of 8 can be decomposed into five iterations and two iterations. The partial products, 40 and 16, compose the total of 56: $7 \times 8 = (5 \times 8) + (2 \times 8) = 40 + 16 = 56$.

7 iterations of the unit of 8, decomposed into 5 and 2 iterations



The unit stays the same. The iterations are decomposed.

Questions to ask:

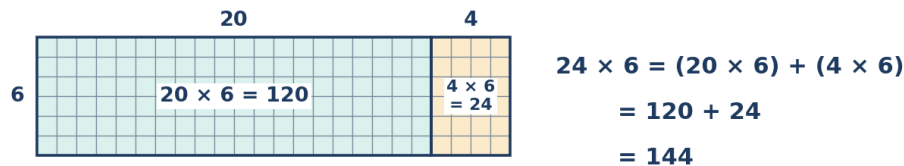
“What stayed the same? What changed?”

“How do the two partial products compose the total?”

“Which other decomposition of 7 would also work?”

Example 3 • place value and the algorithm: 24×6

Decomposing 24 into 20 and 4 partitions the area model into 20×6 and 4×6 . The partial products, 120 and 24, compose 144. In the standard algorithm, 6×4 ones produces 24 ones. These are decomposed into 2 tens and 4 ones. Multiplying 6 by 2 tens produces 12 tens. Composing those 12 tens with the 2 regrouped tens gives 14 tens, or 140. Together with the 4 ones, the product is 144. Connect each step to the partial products.



The partial areas compose the total area.

Decomposing 24 by place value into 20 and 4.

Questions to ask:

“What does the 2 in 24 represent?”

“Where is 20×6 in the algorithm?”

“How does the distributive property explain the partial products?”

Example 4 • changing the unit or the iterations

Doubling the number of iterations, or doubling the unit, doubles the total.

Expression	Iterations	Unit	Total
3×4	3	4	12
6×4	6	4	24
3×8	3	8	24

Questions to ask:

“What happens to the total when we double the unit but keep the number of iterations the same? What happens when we double the iterations but keep the unit the same?”

Key Idea • Bar models, area models, partial products, and the algorithm represent the same relationships. Connect them explicitly; they are not a required sequence.

What Student Responses Can Show

Student responses provide evidence of their mathematical thinking. When a response is unclear, ask students to explain their strategy or show another representation.

Student response	What to investigate	Possible teacher response
Reads 3×4 as $3 + 4$	Whether the student identifies a unit and its iterations	Ask the student to represent 3×4 in a bar model and identify the unit and number of iterations
Finds 7×8 by counting by ones	Whether the student uses a known fact to compose the total	Ask how 5×8 could help
Writes 1,224 for 24×6	Whether the student understands the value of each partial product and how those quantities are composed	Ask the student to explain how the answer was obtained. Use the area model to compare the partial products 120 and 24 and how they compose 144
Says 6×4 doubles 3×4 but cannot explain why	Whether the student connects the iterations to the total	Ask the student to show both in a bar model

Questions About Mathematical Relationships

Questions about answers are important. Also ask students to explain the units, quantities, and relationships behind their work.

Instead of only	Also ask
“What is 7×8 ?”	“What is the unit, and how many times is it iterated?”
“Use the algorithm.”	“Where are the partial products in the algorithm?”
“Draw a model.”	“How does your model show the same relationship as the equation?”
“Is that right?”	“How could you decompose the problem to check it?”

Across the Grades

Across grade levels, the numbers and representations change. The relationship among unit, iterations, and total stays the same.

Grade band	Focus	Example	Representation
Grades 2–3	Iterating a unit	$3 \times 4 = 12$	Bar model
Grades 3–4	Decomposing iterations to derive facts	$7 \times 8 = 40 + 16$	Bar or area model
Grades 4–5	Place-value decomposition and the algorithm	$24 \times 6 = 120 + 24$	Area model, partial products
Grades 6–8	Generalizing the distributive property	$3(x + 4) = 3x + 12$	Area model, equations

For Coaches

Coaches can work with teachers to examine how instructional tasks represent units, iteration, and decomposition. Use student work to investigate how students connect these relationships to multiplication procedures and identify what instruction they may need next.

Questions for a coaching conversation:

- Did students name the unit and the number of iterations?
- Where did students decompose the unit or the iterations?
- Where did instruction connect partial products to the algorithm?
- What do the student responses suggest for the next lesson?

For School Leaders

School leaders can support this work by providing curricular resources that connect mathematical representations and procedures, time for teachers to examine student work, and assessments that address conceptual understanding, procedural fluency, and mathematical reasoning.

Read DMT Insight 21 at mathsuccess.io/dmtinsights.