

DMT COMPANION

WHY MEASUREMENT MATTERS: HOW THE UNIT GIVES A NUMBER ITS MEANING

How to Use This

This Companion accompanies DMT Insight 18 for educators. Answer the three guiding questions before the Insight or video, and then answer them again afterward.

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. When a student reports a length of 9, can the student name the unit the 9 counts?
2. If the unit gets larger, can the student predict how the count changes, and explain why?
3. On a ruler, does the student count the units between the ends, or read the last numeral?

Before and After: One Book, Two Counts

Students need to match the unit size with the number of units. The situation stays the same: one book, measured with paper clips and with craft sticks. Only the reasoning changes.

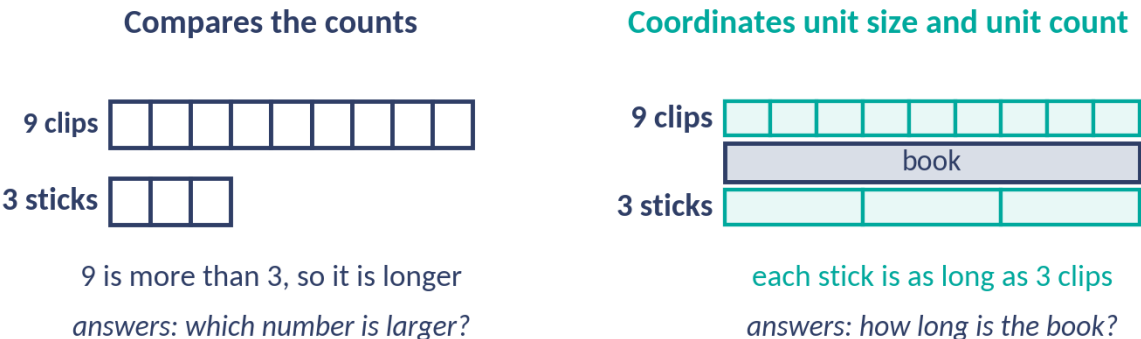


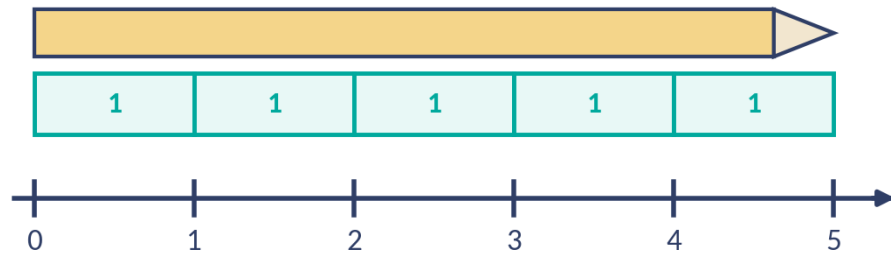
Figure 1. Left: the counts 9 and 3 compared as numbers, as if they measured in the same unit. Right: the same book is 9 paper clips long and 3 craft sticks long, because each stick is as long as 3 clips.

The takeaway: both counts can be correct. The unit tells us what each number means.

Worked Examples

Each example uses a model and teacher questions to keep attention on the unit and the count, not the procedure.

Example 1 · Iterating a unit: building a ruler



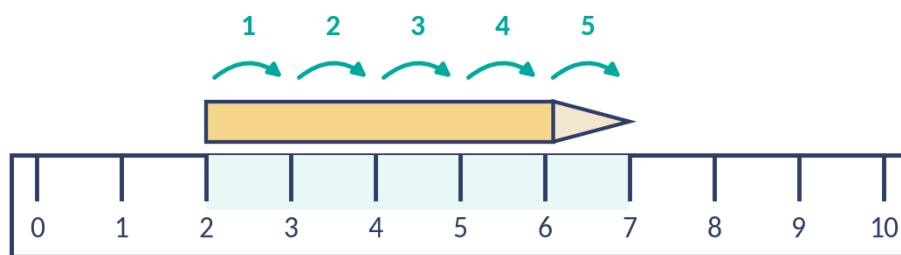
5 iterations of the unit compose the length of the pencil

Figure 2. One cube unit iterated 5 times along the pencil, with no gaps or overlaps. Numbering the end of each iteration from 0 turns the row of units into a ruler.

Students place one unit at the end of the pencil, mark where it ends, and move the same unit forward from that mark. The number of iterations tells how many units compose the length. Recording the end of each iteration on a number line shows that each numeral counts the units from 0.

- “What is the unit, and where did you start?”
- “Are there any gaps or overlaps between the units?”
- “What does the 5 count?”

Example 2 · Reading a ruler as iterated units



count the intervals: 5 units, not 7

Figure 3. The pencil starts at 2 and ends at 7. Counting the 5 intervals between its ends gives a length of 5 units.

When an object does not start at 0, reading the numeral at its end gives the wrong length. Students count the intervals between the two ends instead. Each interval is one unit. The numeral tells how many units are between 0 and that mark.

- “Where does the pencil start, and where does it end?”
- “How many units are between the two ends?”
- “Why is the length not 7?”

Example 3 · Partitioning the unit of 1

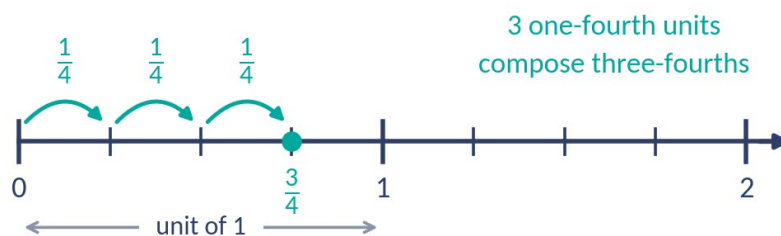


Figure 4. The unit of 1 is partitioned into 4 equal-sized units. Three one-fourth units, iterated from 0, compose three-fourths.

When the unit is too large to measure a length exactly, students partition the unit into equal-sized units and count the new units. The structure is the same as in Example 1, but the unit is now smaller than 1. The denominator names the unit size, and the numerator counts how many of those units make up the length.

- “What is the unit of 1 here?”
- “Into how many equal-sized units is it partitioned?”
- “How many one-fourth units make up this length?”

Language That Makes the Structure Visible

Keep this by the board. The language on the right keeps attention on the unit and what you're counting.

Instead of	Say
Line it up and read the number.	Count the units from one end of the object to the other.
It is 9 long.	It is 9 paper clips long: 9 iterations of the paper-clip unit.
Big units and little units	Unit size and unit count: a larger unit gives a smaller count for the same length.
Measure with your hands.	Choose one unit, and iterate it with no gaps or overlaps.
Read the tick mark.	Count the intervals between the marks. Each interval is one unit.
Cut it into pieces.	Partition the unit of 1 into equal-sized units.

What You Should See and Hear

- Students naming the attribute before they measure.
- Units iterated end to end, with no gaps or overlaps.
- Students predicting how the count will change when the unit changes, and explaining why.
- Students counting intervals on a ruler, including when an object does not start at 0.
- Measurements reported with the unit named, not only the number.

Across the Grades

Band	What students build	Model	What changes
Grades K–1	Compare lengths directly and indirectly, then iterate one unit	Objects side by side, string, cubes end to end	From which is longer to how many units long
Grades 2–3	Coordinate unit size with unit count, and read a ruler as iterated units	Ruler, number line	From reading the last numeral to counting intervals from 0
Grades 3–5	Partition the unit of 1, and compose units of different sizes	Number line, bar model, and metric units	From whole units to fractions and tenths of a unit

Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at mathsuccess.io/dmtinsights. The Newsletter for this issue gives a one-page version to share with families and administrators. For area, volume, and angle, see DMT Insight 56, Geometric Measurement: Before the Formulas.