DMT COMPANION

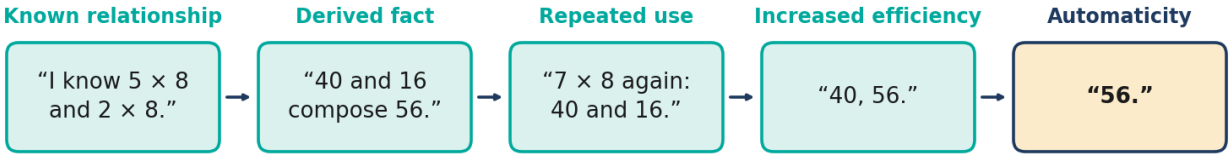
Insight 16 · 2025 · for teachers, coaches, principals, and families

BEYOND DRILL AND PRACTICE: BUILDING FACTS FROM RELATIONSHIPS

How to Use This

This Companion is for anyone working with DMT Insight 16. The Insight reports a DMTI study that compared strategy-based instruction with traditional drill and practice. This Companion shows what building facts from relationships looks and sounds like in a classroom and at home.

Students need facts they can access efficiently. Mathematical relationships give students something to reason from, and practice makes those relationships increasingly efficient. Over time, many facts become automatic.



One fact, 7 × 8, at each point in the progression.

> **Key Idea • Practice matters. The question is what mathematical relationships students have available to reason from as they practice.**

Three Guiding Questions

Ask these when a student is working on a fact that is not yet automatic.

1. **What fact does the student already know that could help?**
2. **What mathematical relationship connects the known fact to the unknown fact?**
3. **Is the student's reasoning becoming increasingly efficient?**

Build Facts From Known Relationships

Teach a small set of relationships explicitly and use the same language each time: unit, iterate, compose, decompose. Each example follows the same progression: a known fact, a relationship, a derived fact, and practice toward efficiency. A model earns its place when it makes the relationship visible.

Example 1 · doubles and near doubles: 8 + 7



8 7, one less than 8

Known fact: 8 + 8 = 16. Relationship: 8 + 7 is one less than 8 + 8. Derived fact: 8 + 7 = 15.

The language to use: *"What double do you know that could help?"*

Practice toward efficiency: *return to 6 + 7, 7 + 8, and 9 + 8 until using the double takes one step.*

Example 2 · make ten: 8 + 6



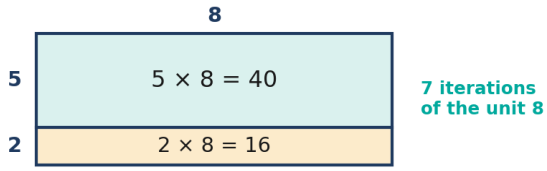
8 and 2 compose 10 4 more

Known fact: 8 + 2 = 10. Relationship: decompose 6 into 2 and 4, and compose 8 and 2 to make the benchmark 10. Derived fact: 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14.

The language to use: *"How could you use 10 to build this fact?"*

Practice toward efficiency: *return to 9 + 5, 8 + 5, and 7 + 6 until composing 10 takes one step.*

Example 3 · decompose the number of iterations: 7 × 8

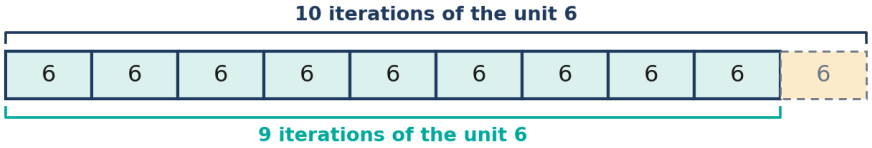


Known facts: 5 × 8 = 40 and 2 × 8 = 16. Relationship: the unit is 8. Decompose the seven iterations into five iterations and two iterations. Derived fact: 7 × 8 = 5 × 8 + 2 × 8 = 56.

The language to use: *"What multiplication fact do you already know that could help?"*

Practice toward efficiency: *return to 6 × 8, 7 × 6, and 7 × 7, each built from a ×5 fact.*

Example 4 · compensation: 9 × 6

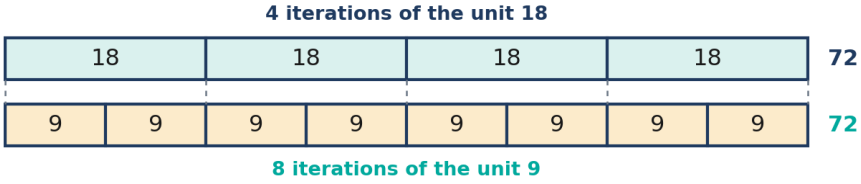


Known fact: 10 × 6 = 60. Relationship: ten iterations of the unit 6 compose 60. Nine iterations are one unit of 6 less. Derived fact: 9 × 6 = 10 × 6 − 6 = 54.

The language to use: *"How could a ×10 fact help?"*

Practice toward efficiency: *return to 9 × 7, 9 × 8, and 9 × 4, each built from a ×10 fact.*

Example 5 · doubling and halving: 4 × 18



Known fact: 8 × 9 = 72. Relationship: partition each unit of 18 into two units of 9. The number of iterations doubles, and the unit is halved. Derived fact: 4 × 18 = 8 × 9 = 72.

The language to use: *"What changed? What stayed the same? Why is the product equivalent?"*

Practice toward efficiency: *try 4 × 16 = 8 × 8 and 5 × 14 = 10 × 7. Ask which form takes fewer steps.*

Key Idea • The derived fact is not the end point. With repeated use, many facts become automatic.

Practice for Reasoning and Retrieval

Practice should give students opportunities to retrieve facts efficiently and to use relationships when recall is not immediate. Over time, students should need fewer steps and retrieve more facts automatically.

Timing can show efficiency, but speed alone does not tell us what a student understands or what the student can do when recall fails.

- **Number talks.** Pose one fact or problem, such as 18 × 5, and record the strategies students use. The purpose is not to collect as many strategies as possible. Ask students to compare strategies and identify the mathematical relationship they share.
- **Games.** Games can provide repeated retrieval without presenting facts in the same order every time. They are most useful when students still have to attend to the relationship among the facts.
- **Mixed and spaced practice.** Mix related facts and operations within a practice set, and return to the same relationships across the week in short warm-ups.

Key Idea • Practice should include efficient retrieval and reasoning from relationships.

Practice at Home

At home, practice can reinforce the same relationships students use at school. Rather than supplying the answer immediately, ask: “What fact do you already know that could help?” Then give students opportunities to retrieve and practice the fact again.

- **Play games.** Card and dice games give repeated practice with facts in a different order each time.
- **At the store.** “If apples are $12 a bag, how much are 4 bags? What fact could help?”
- **In the kitchen.** “My recipe makes 8 muffins, and I need 56. How many recipes? What fact could help?”
- **Ask the teacher** which relationships the class is using, and use the same language at home.

The Language Shift

Keep this by the board. Each question on the right asks for a known fact or a relationship.

Instead of	Ask
“Just memorize it.”	“What fact do you already know that could help?”
“Start counting from the beginning.”	“Is there a known fact or benchmark you could use?”
“Try another strategy.”	“What relationship do you see?”
“Go faster.”	“How could you do that in fewer steps?”
“That’s wrong.”	“Show me what relationship you used.”

The goal is not for students to use a strategy for every fact forever. The goal is increasingly efficient access to connected facts.

> **Key Idea • Build the relationship. Practice it. Make it increasingly efficient.**

Where to Go Next

Read DMT Insight 16, Beyond Drill and Practice: Building Multiplication Fact Fluency, for the study behind this Companion. Find it at mathsuccess.io/dmtinsights, along with the rest of the DMT Insights series.

DMTI resources for fact practice include strategy cards, fluency probes, and fluency apps (Math Fluency with Sports: Addition and Subtraction; Math Fluency with Sports: Multiplication and Division; Picnic Party; Dino Dice). Find them at dmtinstitute.com.

Brendefur, J., & Strother, S. (2021). *Math facts: Kids need them. Here’s how to teach them.* Developing Mathematical Thinking Institute.