DMT COMPANION

Insight 15 · 2025 · for teachers, coaches, principals, and families

Math Success by DMTI

FRACTION UNDERSTANDING: CHALLENGES, MISCONCEPTIONS, AND EFFECTIVE PRACTICES

How to Use This

This Companion accompanies DMT Insight 15 for educators. Work the three guiding questions before the Insight or the video, and again afterward.

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. When a student writes $\frac{1}{2}+\frac{1}{3}$ as $\frac{2}{5}$, what is the student operating on?
2. Does the student focus on the numerals, or on the count and size of the fractional units?
3. Can the student name the unit of 1 in this task and partition it into equal units?

Before and After: Two Numbers or One Quantity

The shift is from reading a fraction as two whole numbers to reading it as a count of units of a named size. Before, the student adds across. Afterward, the student renames both fractions in units of the same size and estimates first.

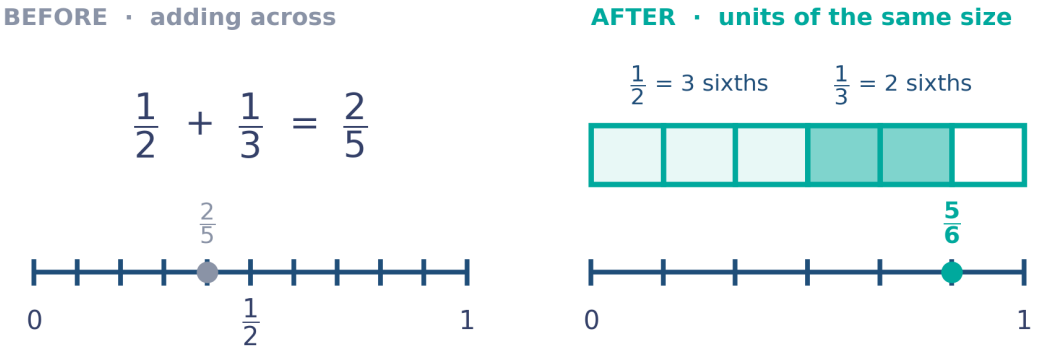

Figure 1. Left: adding across gives $\frac{2}{5}$, less than the $\frac{1}{2}$ the student started with. Right: in sixths, $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$.

The takeaway: both students produce an answer. Only one of them can tell whether it is reasonable before checking.

Worked Examples

Each example pairs a model with the language that keeps attention on the unit size and the count.

Example 1 · Partitioning and iterating

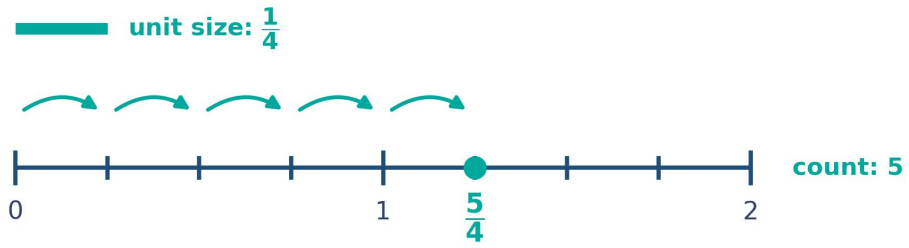

Figure 2. The space from 0 to 1 is partitioned into four equal units, and $\frac{1}{4}$ is iterated five times to reach $\frac{5}{4}$.

The denominator names the unit size, and the numerator counts those units. This language makes a fraction greater than 1 ordinary.

- "Partition the space from 0 to 1 into four equal units. What is each unit called?"
- "Iterate $\frac{1}{4}$ five times. Where are you, and how do you know?"
- "How many fourths compose 1, and how many more reach $\frac{5}{4}$?"

Example 2 · Equivalence

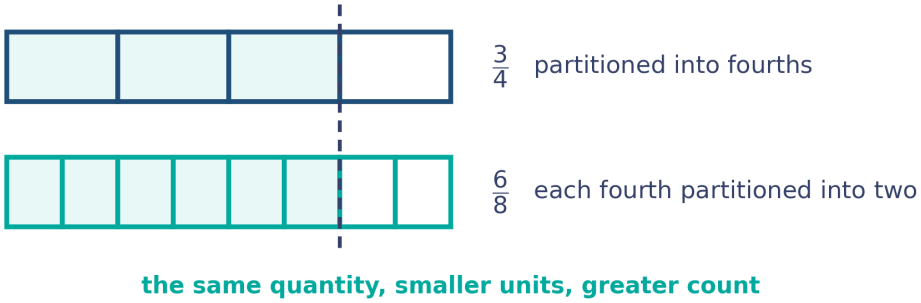

Figure 3. Each fourth is partitioned into two equal units. The shaded quantity is both $\frac{3}{4}$ and $\frac{6}{8}$.

Equivalence renames the same quantity using smaller units and a greater count. Multiplying the numerator and denominator by 2 describes what the partitioning already shows.

- "Partition each fourth into two equal units. How many units are there now?"
- "How many of those units are shaded? Did the quantity change?"
- "What happened to the unit size, and what happened to the count?"

Example 3 · Decomposing and composing to add

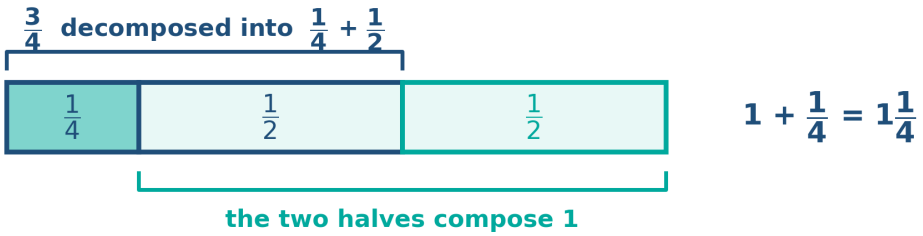

Figure 4. $\frac{3}{4}$ is decomposed into $\frac{1}{4}$ and $\frac{1}{2}$. The two halves compose 1, so $\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$.

Flexible operating stems from taking fractions apart and putting them back together. The same work shows why the units must be the same size before they are combined.

- "Can you decompose $\frac{3}{4}$ into quantities that help?"
- "Which two quantities compose 1?"
- "How much is left over, and what is the total?"

The Language Shift

Keep this by the board. The words on the right make the reasoning explicit.

Instead of	Say
Numerator and denominator.	The numerator is the count, and the denominator is the unit size.
Out of the whole.	Out of the unit of 1, partitioned into equal units.
Three out of four.	Three units, each $\frac{1}{4}$.
Find a common denominator.	Rename both in units of the same size.
Multiply top and bottom.	Partition each unit again. Same quantity, smaller units, greater count.

What to look for in a classroom

- Students naming the unit of 1 before they partition.
- Fractions greater than 1 placed on a number line without hesitation.
- Students estimating against 0, $\frac{1}{2}$, and 1 before computing.
- Equivalent fractions explained as the same quantity in smaller units.
- Bar models and number lines used before area models.

Across the Grades

Band	What students build	Model	What changes in their thinking
3	Partition a ribbon into four equal units. Show $\frac{3}{4}$. What is the unit of 1 here?	Bar model, number line	From equal parts of a shape to a named unit that can be counted.
4–5	Place $\frac{5}{4}$ on a number line from 0 to 2. Show why $\frac{6}{8}$ is $\frac{3}{4}$.	Number line, area model	From fractions less than 1 to quantity and equivalence.
6	Add $\frac{1}{2}$ and $\frac{1}{3}$. Estimate first. Why does the sum exceed $\frac{1}{2}$?	Bar model, number line	From a procedure to reasoning about size.

Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at mathsuccess.io/dmtinsights. The Newsletter for this issue gives a one-page version for families and for staff meetings. For more classroom materials, visit mathsuccess.io.