

DMT COMPANION

Insight 14 · 2025 · for teachers, coaches, principals, and families



MATH FACT FLUENCY: BUILDING FACTS FROM RELATIONSHIPS

How to Use This

This Companion is for anyone working with DMT Insight 14. Start with the three questions that follow. Answer them before you read the Insight or watch the video, then come back and answer them again. Coaches can open a team meeting with the same three questions, and use the worked examples to keep the conversation close to the classroom.

Key Idea • Answer the three questions first, then read. The gap between your two answers is the point of the Companion.

Three Guiding Questions

Answer these before you read the Insight or watch the video, then again afterward.

1. When a student forgets a fact, what does the student have to rebuild it from?
2. When your students practice facts, are they also using and strengthening relationships among those facts?
3. Are your students' strategies taking fewer steps over time, or is every fact reconstructed each time?

Before and After: The Shift This Insight Asks For

The shift is from treating each fact as an isolated item to building unknown facts from known relationships. In the before, $8 + 7$ is treated as one item among many. If the fact is not recalled, the student may have little to reason from. In the after, the student reasons from $8 + 8 = 16$ to $8 + 7 = 15$.

BEFORE • an isolated fact

$$8 + 7 = ?$$

"I forgot that one."

The fact is one item among many.
The student may have little to reason from.

AFTER • a fact built from a known relationship

$$8 + 8 = 16 \xrightarrow{\text{one less}} 8 + 7 = 15$$

"8 + 8 is 16, so 8 + 7 is 15."

The student reasons from a known fact.
A forgotten fact can be rebuilt.

Before: an isolated fact. After: a fact built from a known relationship.

Fluency is more than speed. A fluent student can recall a fact or derive it efficiently from a known relationship. With practice, useful relationships become increasingly efficient, and many facts become automatic.

Key Idea • If a fact is forgotten, understanding gives the student somewhere to rebuild it from.

Worked Examples: The Model and the Words to Use

Example 1 • use a known fact: $8 + 7$ from a double

$8 + 7$: one less than $8 + 8$



$8 + 8 = 16$, so $8 + 7 = 15$

Two eights compose 16. $8 + 7$ is one less than $8 + 8$, so $8 + 7 = 15$.

The language to use:

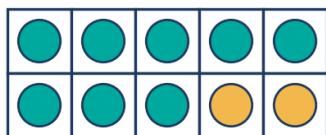
"What fact do you already know that could help?"

"How is $8 + 7$ different from $8 + 8$?"

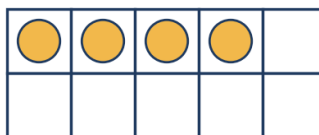
"Which other facts are one more or one less than a double?"

Example 2 · compose and decompose: $8 + 6$ by making ten

$8 + 6$: make ten, then add 4



8 and 2 compose 10



4 more

$$8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$$

Decompose 6 into 2 and 4. 8 and 2 compose 10, and $10 + 4$ is 14.

The language to use:

“How many more does 8 need to make 10?”

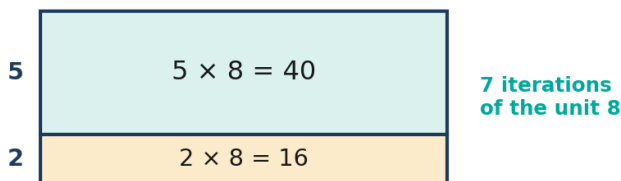
“How did you decompose the 6?”

“Which other facts could you build from 10?”

Example 3 · decompose the number of iterations: 7×8 from 5×8 and 2×8

7×8 : five eights and two eights

8



$$7 \times 8 = 5 \times 8 + 2 \times 8 = 40 + 16 = 56$$

The unit is 8. Seven tells how many times the unit is iterated. Decompose the seven iterations into five iterations and two iterations. Five eights and two eights compose seven eights.

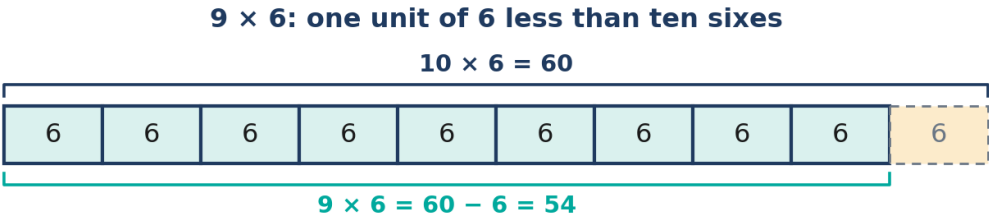
The language to use:

“What is the unit? How many times is it iterated?”

“Which multiplication fact do you already know that could help?”

“How does 5×8 help with 6×8 or 7×8 ?”

Example 4 · use a known relationship: 9×6 from 10×6

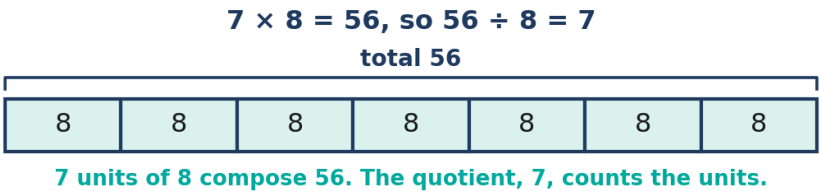


The unit is 6. Ten iterations compose 60. Nine iterations are one unit of 6 less, so $9 \times 6 = 54$.

The language to use:

- “What is ten sixes?”*
- “How is nine sixes different from ten sixes?”*
- “Which other facts could you build from a $\times 10$ fact?”*

Example 5 · use an inverse relationship: $56 \div 8$ from 7×8



If $7 \times 8 = 56$, then $56 \div 8 = 7$. A known multiplication fact gives the division fact.

The language to use:

- “What multiplication fact could help?”*
- “What does the quotient count?”*

Key Idea · A derived-fact strategy is not another rule to memorize. It is a relationship students can reuse.

The Language Shift

Keep this by the board. The right column asks for the relationship.

Instead of	Say
Just memorize it.	What fact do you already know that could help?
Use the doubles trick.	$8 + 8$ is 16. How is $8 + 7$ related to it?
Start counting from the beginning.	Is there a fact or benchmark you could use instead?
You should know this by now.	What do you know that is close to it?
Go faster.	How could you do that in fewer steps?
Practice your facts.	Use a relationship to derive it, then practice until the fact or relationship becomes efficient.

What to look for in a classroom:

- Students naming the known fact they used.
- A small number of models used repeatedly: ten-frames, number lines, and area models.
- Practice that includes related facts as well as opportunities for efficient retrieval.
- Strategies taking fewer steps over time, and more facts recalled automatically.
- Students moving between derivation and recall rather than being required to explain a strategy for every fact.

Key Idea • Practice matters. It should make useful relationships increasingly efficient while helping more facts become automatic.

Across the Grades

The role of fact fluency changes across the grades, but the underlying progression remains: known relationship → derived fact → repeated use → increased efficiency → automaticity.

Grade band	Example	Model	The shift it asks for
Grades K-2	Doubles, near doubles, and making ten for addition and subtraction facts	Ten-frame, number line	From counting-based strategies toward reasoning from known facts and relationships
Grades 3-5	Multiplication facts built from facts with 2, 5, and 10, and division facts built from multiplication	Area model, bar model, ratio table	From isolated multiplication facts toward reasoning about the unit and the number of iterations, and using known multiplication relationships to derive division facts
Grades 6-8	Using automatic and derived facts within calculations involving fractions, decimals, ratios, and proportional relationships	Ratio table, double number line	From developing basic fact fluency to using that fluency efficiently within more complex mathematics

Key Idea • This is not a rigid sequence. Students move back and forth between reasoning and retrieval as new facts and operations are introduced.

Where to Go Next

Read DMT Insight 14, Math Fact Fluency: Building Facts From Relationships, and watch the short video. Find both at mathsuccess.io/dmtinsights, along with the rest of the DMT Insights series.

Try this every day for a week. Write one fact on the board, such as 7×8 , and ask: “What fact do you already know that could help?” Record the different relationships students use. Then ask which strategy is most efficient and why.

Over the week, look for whether students begin using fewer steps and retrieving more facts automatically.