

# DMT COMPANION

Insight 10 · 2026 · for teachers, coaches, principals, and families



## THE POWER OF DIVISION: UNDERSTANDING WHAT THE QUOTIENT COUNTS

### How to Use This

This Companion accompanies DMT Insight 10 for educators. Work the three guiding questions before the Insight or the video, and again afterward.

### Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. When a student writes  $18 \div 3 = 6$ , can the student determine from the situation whether 3 represents the unit size or the number of units?
2. Can the student write a sharing situation and a measuring situation for one equation?
3. Can the student say what the quotient counts, or only that the procedure produced it?

### Before and After: One Equation, Two Questions

Students need to read the situation before applying a procedure. The numbers stay the same: 18 and 3. The situation determines what the 3 represents and what the quotient counts.

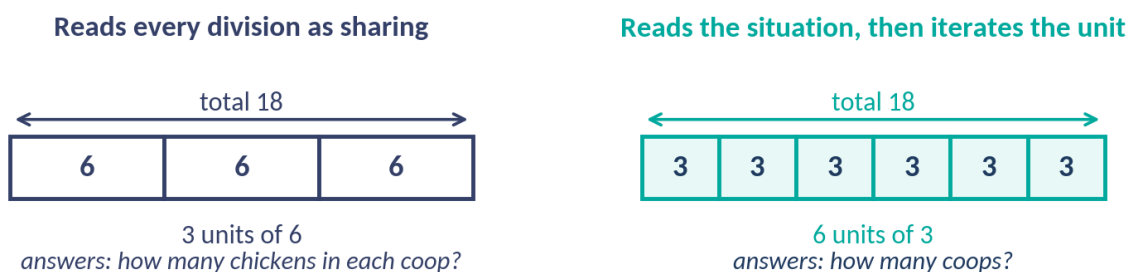


Figure 1. The coops hold 3 chickens each. Left: the total partitioned into 3 equal units, answering a question the situation did not ask. Right: the unit of 3 iterated 6 times, so the 6 counts coops.

**The takeaway: both students write  $18 \div 3 = 6$ . Only one can say what the 6 counts.**

## Worked Examples

Each example pairs a model with the questions a teacher would ask to keep attention on the structure rather than on the next step of the procedure.

### Example 1 • Measurement division: the situation gives the unit

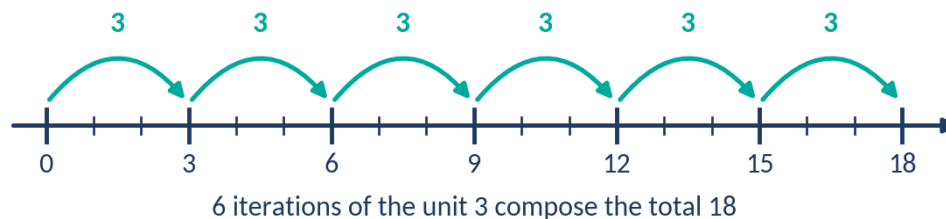


Figure 2. The unit 3 is iterated along the number line from 0. Six iterations compose the total 18, so the farmer needs 6 coops.

The situation gives the unit (3 chickens in each coop) and asks how many of those units make up the total. A number line makes the iterations countable, and it keeps the connection to multiplication in view, because 6 iterations of 3 compose 18.

- “What is the unit here, and how do you know?”
- “How many iterations of 3 compose 18?”
- “What does the 6 count, chickens or coops?”

### Example 2 • Partitive division: the situation gives the iterator



Figure 3. The total of 18 is one bar model partitioned into 3 equal parts, with each part representing one unit. Each unit is 6, so each coop holds 6 chickens.

The situation gives the iterator (3 coops) and asks how large each equal unit is. The iterator is 3. The unknown is the size of each unit. A bar model makes the partitioning visible. The numbers are the same as in Example 1, but the quotient counts something different.

- “Into how many equal units is the total partitioned?”
- “How large is each unit?”
- “What does the 6 count this time?”

### Example 3 · Chunking toward the standard algorithm

|            |   |    |    |     |
|------------|---|----|----|-----|
| units of 8 | 1 | 10 | 8  | 18  |
| total      | 8 | 80 | 64 | 144 |

10 units of 8 compose 80, and 8 more compose 64.  
80 and 64 compose 144, so the iterator is 18.

Figure 4. A ratio table records chunks of the unit 8 on the way to  $144 \div 8$ .

For  $144 \div 8$ , students take several units at a time rather than iterating one at a time. A ratio table holds the chunks and keeps the relationship between the iterator and the total in view. The standard algorithm records the same work more compactly, so the written procedure connects back to relationships the student already understands.

- “What is a chunk of 8 that you already know?”
- “How many units of 8 have you used, and how much of the total is left?”
- “Where do you see that step in the standard algorithm?”

### Language That Makes the Structure Visible

Keep this by the board—the words on the right name the structure the familiar phrase abbreviates.

| Instead of                 | Say                                                                                                                                           |
|----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| Goes into                  | How many units of 3 compose 18?                                                                                                               |
| The number you divide by   | The unit, when the situation gives the unit size; the iterator, when it gives how many units there are                                        |
| Bring down the next digit. | Rename or decompose the remaining quantity using the next smaller place-value unit, then continue dividing.                                   |
| Remainder 2                | 2 of the total is left over, less than one more complete unit                                                                                 |
| Flip and multiply          | Dividing by two-thirds asks how many two-thirds units compose the total. Multiplying by the reciprocal records that relationship efficiently. |

### What You Should See and Hear

- Students naming the total, the unit, and the iterator before they compute.
- The same equation modeled as a sharing situation and as a measuring situation.
- Number lines used for measuring, and bar models used for sharing.
- Students saying what the quotient counts, not only what it equals.
- Students connecting chunking and partial quotients to the standard algorithm.

## Across the Grades

| Band       | What students build                                                                    | Model                              | What changes                                                           |
|------------|----------------------------------------------------------------------------------------|------------------------------------|------------------------------------------------------------------------|
| Grades 2–3 | Experience both sharing and measuring situations                                       | Bar model, equal groups            | From dealing out one at a time to iterating a unit                     |
| Grades 3–4 | Name and compare the two structures explicitly, and connect division to multiplication | Number line, array, and area model | From counting the result to predicting it from the factor relationship |
| Grades 5–6 | Extend the same structure to fractions, decimals, and rates                            | Double number line, ratio table    | From whole-number units to fractional and decimal units                |

## Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at [mathsuccess.io/dmtinsights](https://mathsuccess.io/dmtinsights). The Newsletter for this issue gives a one-page version to share with families and administrators.