

DMT COMPANION

Insight 5 · 2024 · for teachers, coaches, principals, and families



HARNESSING MISCONCEPTIONS

How to Use This

This Companion is for anyone working with DMT Insight 5. Start with the three questions that follow. Answer them before you read the Insight or watch the video, then come back and answer them again. Coaches can open a team meeting with the same three questions, and use the worked examples to keep the conversation close to the classroom.

Key Idea • Answer the three questions first, then read. The gap between your two answers is the point of the Companion.

Three Guiding Questions

Answer these before you read the Insight or watch the video, then again afterward.

1. What idea or conception is the student using that makes this answer reasonable to them?
2. Where does that idea work mathematically, and what has changed in this situation?
3. What model, representation, or example could help the student see the boundary of the old idea and build a more complete one?

Mistake or Misconception?

Mistake	Misconception
The student knows the underlying relationship but makes an execution error.	The student is reasoning from an underlying idea that does not apply in this situation.
Often inconsistent.	Often systematic and repeatable.
Student may self-correct when prompted.	Student may defend the answer because it makes sense.
Usually easier to address.	Requires the underlying conception to be reorganized.

Before correcting a wrong answer, determine which one you are looking at.

Before and After: The Shift This Insight Asks For

The shift is in what a teacher does with a wrong answer. In the before, the teacher replaces the answer with the correct one and moves on, leaving the idea that produced it intact. In the after, the teacher surfaces that

idea, shows where it holds, and shows where it no longer applies. Comparing 0.45 and 0.7 makes the difference plain.

BEFORE • replace the answer

$$\del{0.45 > 0.7}$$

$$0.7 > 0.45$$

"No, 0.7 is bigger." The rule survives.

AFTER • find where the rule came from



"45 is more than 7" counts the digits.
The line asks how far along the unit each one sits.

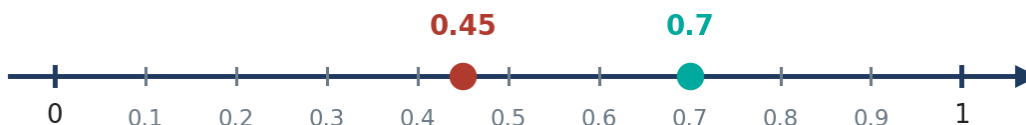
Before: the answer is corrected and the rule survives. After: the number line asks how far along the unit each number sits, which is the question the rule ignored.

A student who says 0.45 is greater than 0.7 may be reasoning that 45 is greater than 7. That comparison is correct for whole numbers, but it ignores the decimal units. The idea is reasonable; it is being used where it no longer applies.

Key Idea • A misconception is a reasonable idea used where it no longer applies. Find where it works, and you have found what to reorganize.

Worked Examples: The Model and the Words to Use

Example 1 • 0.45 and 0.7 on a number line



0.45 sits between 0.4 and 0.5. 0.7 sits further along.

One unit of one, partitioned into tenths. Position on the line settles the comparison and shows why counting digits does not.

Conception: 45 is greater than 7. Where it works: comparing whole numbers. Where it breaks: in decimal notation, the digits count different units. New relationship: 0.7 is seven tenths, or seventy hundredths, and seventy hundredths is greater than forty-five hundredths.

The language to use:

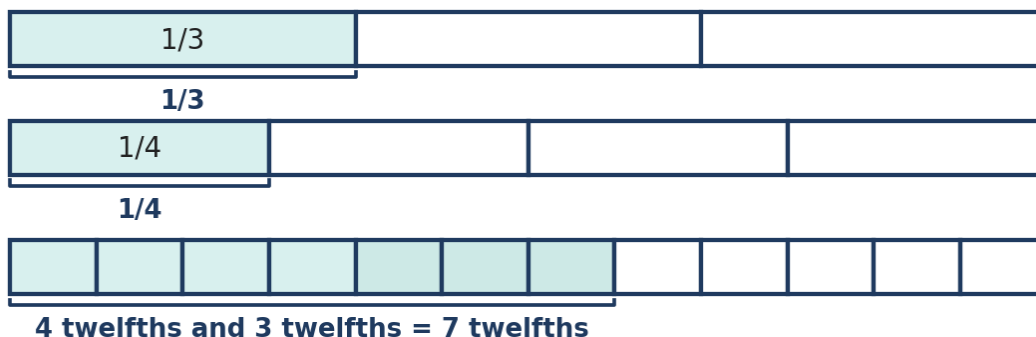
"You said 0.45 is bigger because 45 is bigger than 7. When is that true?"

"Put both numbers on this line. Where does 0.45 land? Where does 0.7 land?"

"0.45 is between four tenths and five tenths. 0.7 is seven tenths. Which is further from zero?"

Example 2 · one third and one fourth, added.

Thirds and fourths are different units. Twelfths hold both.



Thirds and fourths are different units. Twelfths hold both, making addition possible.

Conception: addition combines the numbers being counted. Where it works: three ones and four ones make seven ones. Where it breaks: thirds and fourths are different-sized units. New relationship: partition both quantities into a common unit, twelfths, before composing them, which gives four twelfths and three twelfths, or seven twelfths.

The language to use:

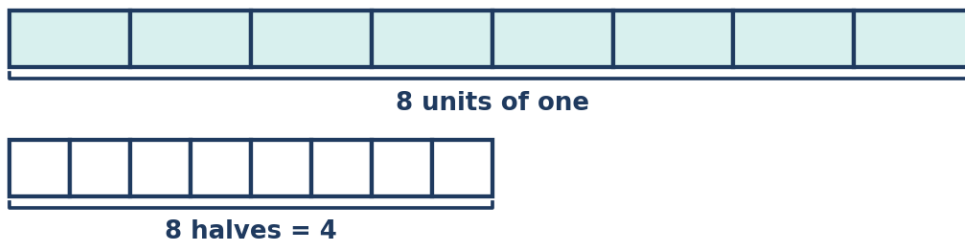
"You added the tops and added the bottoms and got two sevenths. Show me two sevenths on the bar. Is it bigger or smaller than one third alone?"

"One third and one fourth are different sized units. What partition holds both of them?"

"Four twelfths and three twelfths. Now the units match so that we can add them."

Example 3 · multiplying does not always make it bigger (larger).

$8 \times \frac{1}{2}$: the unit is a half, so iterating it eight times gives 4



Eight groups of one-half make four, which is less than eight. Multiplying a positive quantity by a factor less than one produces a result less than the original.

Conception: multiplication makes a quantity larger. Where it works: multiplying a positive quantity by a whole number greater than one. Where it breaks: multiplying by a positive factor less than one. New relationship: the product's size relative to the original quantity depends on the multiplier.

The language to use:

“You expected 8 times a half to be more than 8. When does multiplying make it bigger?”

“What is the unit here? If the unit is smaller than one, what happens to the total?”

“Eight halves is four. Your rule was right for whole numbers, and it stops here.”

Key Idea • Do not correct the answer and move on. Put the student’s rule and the correct reasoning on the same model, and let the comparison do the teaching.

The Language Shift

Keep this by the board. The left column closes an error down. The right column opens it up.

Instead of	Say
That is wrong.	Tell me why that makes sense to you.
Remember the rule.	Where does your idea work? What changed in this problem?
Be more careful.	Would you get the same kind of answer on another problem like this?
Let me show you again.	Show me your reasoning. Now let us put both ideas on the same model.
You have got it now.	Try this new example. Does your original idea still work? Why or why not?

What to look for in a classroom:

- A wrong answer being examined on a model, rather than replaced in a sentence.
- Tasks chosen to surface a known misconception, not just to practice the correct method.
- A misconception revisited weeks later, to check that it was reorganized rather than covered.

Key Idea • The first question after a wrong answer decides whether you learn anything from it.

Across the Grades

Misconceptions look different at each grade, and the move is the same every time: surface the thinking, then confront it with a model.

Grade band	Example	Model	The shift it asks for
Grades K-2	A student reads 19 as larger than 21, because 9 is larger than 1	Bar model, number line	From reading digits to reasoning about the unit
Grades 3-5	A student judges $\frac{1}{8}$ to be larger than $\frac{1}{3}$, because 8 is larger than 3	Bar model, area model	From counting the parts to sizing them

Grades 6-8	A student expects 8×0.5 to be larger than 8	Number line, area model	From a rule about numbers to a rule about units
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Key Idea • Misconceptions cluster around the hard ideas of a topic, which means they mark exactly where the mathematics is being reorganized.

Where to Go Next

Read DMT Insight 5 on harnessing misconceptions, and watch the short video. Find both at dmtinstitute.com/research, along with the rest of the DMT Insights series.

Try this in one lesson this week. Before you teach the topic, write down the error you expect most, then design one task that brings it into the open early enough to work with.