

DMT COMPANION

Insight 4 · 2024 · for teachers, coaches, principals, and families



MATHEMATICAL STRUCTURE

How to Use This

This Companion is for anyone working with DMT Insight 4. Start with the three questions that follow. Answer them before you read the Insight or watch the video, then come back and answer them again. Coaches can open a team meeting with the same three questions and use the worked examples to keep the conversation close to the classroom.

Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. **The words they hear.** Which of these do your students hear every week: unit, compose, decompose, iterate, partition, equal? Which have they never heard you say?
2. **Unit or rule.** When your students meet a new topic, do they look for a unit, or do they wait for a new rule?
3. **The same structure twice.** Name two topics your students think are unrelated. Where does the same structure run through both?

Before and After: The Shift This Insight Asks For

The shift is in what a number is taken to be. In the before, 28 is read primarily as two digits in positions. In the after, 28 is treated as a quantity with structure: two units of ten and eight units of one, 28 units of one, or four units of seven. Which structure is useful depends on the problem.

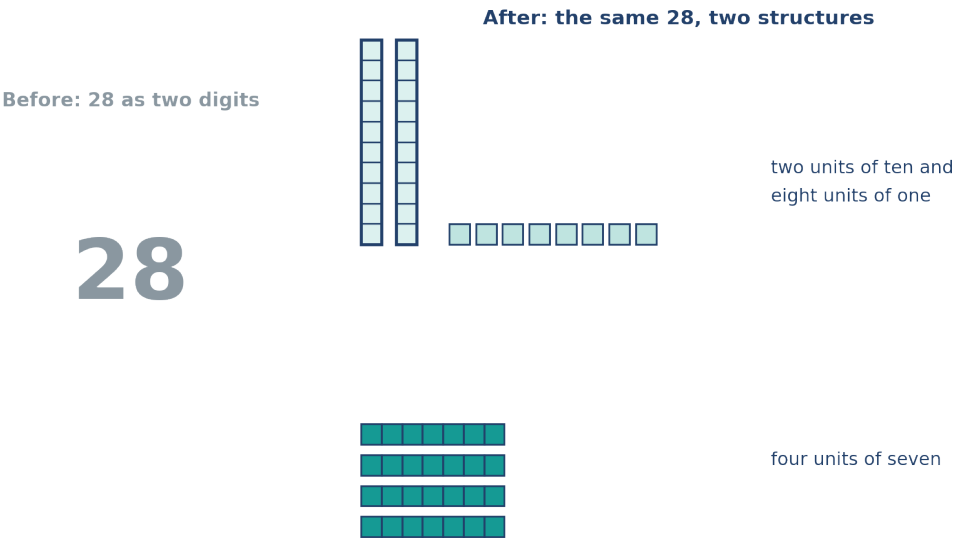


Figure 1. Before, 28 as two digits. After, the same 28 is composed of units of ten and one, and partitioned into four units of seven.

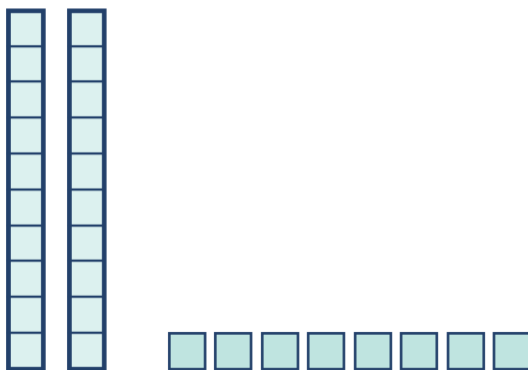
The takeaway: a student who can compose and partition a number has a method for meeting an unfamiliar problem, rather than a gap where a rule should be.

Worked Examples

Each example pairs one model students should reason with and the words to say alongside it. Structural language is not a vocabulary list. Students learn the words by using them to describe what they did.

Example 1 · composing 28 from units

28 = two units of ten and eight units of one



The same 28 is also 28 units of one.

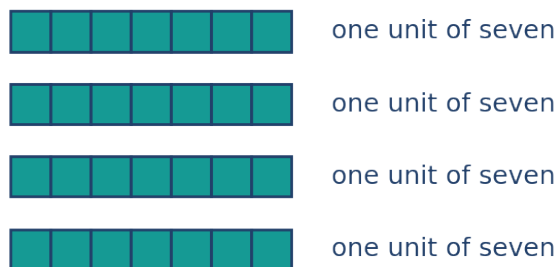
Figure 2. 28 composed from two units of ten and eight units of one. The same 28 is also 28 units of one.

Place value is stated as a composition, not as a column position. Two units of ten and eight units of one compose 28, and the same 28 can be renamed as 28 units of one: $2 \text{ tens} + 8 \text{ ones} = 28 \text{ ones}$.

- “This bar is one unit of ten. How many of them do we have?”
- “Two units of ten and eight units of one. Say the whole thing.”
- “How else could we name 28 using units of one? Twenty-eight units of one.”

Example 2 · partitioning 28 into four sevens

28 partitioned into four units of seven



$$\text{four sevens} = 28 \quad (\text{iterator } 4, \text{ unit } 7)$$

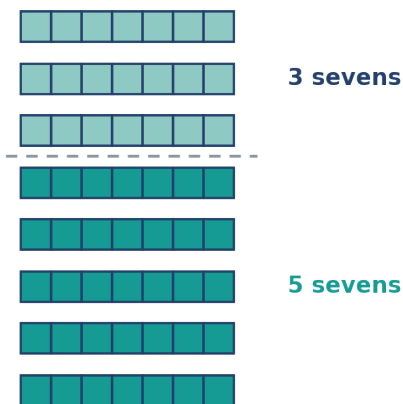
Figure 3. 28 partitioned into four equal units of seven. The unit is seven, and it is iterated four times.

The unit is seven, and the unit is iterated four times. Four tells how many iterations of the unit. Seven tells the size of the unit.

- “We partitioned 28 into four equal parts. What is one part worth?”
- “Four sevens. The four tells how many iterations, and the seven is the unit.”
- “Same 28 as before, partitioned differently. Which partition would you want for sharing among four people?”

Example 3 · eight sevens, decomposed

$$8 \text{ sevens} = 5 \text{ sevens} + 3 \text{ sevens}$$



The unit stays seven. Only the number of iterations changes: $8x = 5x + 3x$

Figure 4. Eight sevens decomposed into five sevens and three sevens. The unit stays seven; only the number of iterations changes.

The unit is seven, and it does not change—only the number of iterations changes. The same structure appears later in algebra: $8x$ can be decomposed into $5x + 3x$.

- “The unit is seven, and it does not change. Only the number of iterations changes.”

- “Five sevens and three sevens. Eight sevens altogether.”
- “You will see the same structure later in algebra. In $8x = 5x + 3x$, x is the repeated quantity.”

The Language Shift

Keep this by the board. The left column names a step. The right column names the structure, and the structure holds in the next topic.

Instead of	Say
Four groups of six.	Four sixes. The unit is six, and we iterate it four times.
Borrow from the tens.	Decompose one unit of ten into ten units of one.
How many groups?	How many iterations of the unit?
Split it into equal parts.	Partition the quantity into equal units.
The whole.	What quantity are we treating as one unit?
What does this number mean?	What unit is this number counting?
Put the numbers together.	Compose the units. What do they make?
That is a different topic.	Same structure. Where is the unit this time?

What to Look For in a Classroom

- The six words spoken by students, not only by the teacher.
- The same model used in two topics students think are unrelated.
- Students asking what the unit is when they meet something new.

Across the Grades

The six structural words run across the grades. The mathematics changes, and the words stay the same. When students hear the same six words at every grade, the topics stop looking unrelated.

Grade band	Example	The shift it asks for
Grades K-2	Compose and decompose base-ten units	From counting individual objects to reasoning with composed units.
Grades 3-5	Partition quantities and decompose products	From isolated facts and procedures to relationships among units and quantities.
Grades 6-8	Equivalent expressions, ratios, and proportional relationships	From a new rule for each topic to recognizing familiar structure in new notation.

Where to Go Next

Read DMT Insight 4, Focusing on Mathematical Structure, and watch the short video on the DMT Insights page at mathsuccess.io/dmtinsights. Try this this week. Pick one of the six words and use it in every lesson for five days, in the mathematics you were already teaching. Do not define the word first. Use it while students are doing the mathematics, and notice which students start using it back.