

DMT COMPANION

Insight 3 · 2024 · for teachers, coaches, principals, and families



PRIORITIZING CONCEPTUAL LEARNING

How to Use This

This Companion accompanies the DMT Insight on prioritizing conceptual learning for teachers, coaches, principals, and families. Start with the three questions that follow. Answer them before you read the Insight or watch the video, then answer them again afterward. Coaches can open a team meeting with the same three questions and use the worked examples to keep the conversation close to the classroom.

Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. **Explain, not just perform.** When your students can carry out a procedure correctly, what would you ask to find out whether they understand why it works?
2. **Fluent but not explained.** Which procedures do your students perform well but cannot explain, and what would you look for to tell the two apart?
3. **Rebuild from the idea.** Can your students show a procedure on a model, so that a forgotten step can be rebuilt from the idea rather than lost?

Before and After: The Shift This Insight Asks For

The shift is in what comes first. In the before, students learn the steps and never name the idea, so the steps have nothing underneath them. In the after, students meet the idea and a model first, and the written steps become a record of something they already understand. Subtraction with regrouping shows the difference plainly.

Before: crossings-out,
nothing underneath

$$\begin{array}{r} 3 1 \\ - 1 7 \\ \hline 2 5 \end{array}$$

After: the decomposition made visible

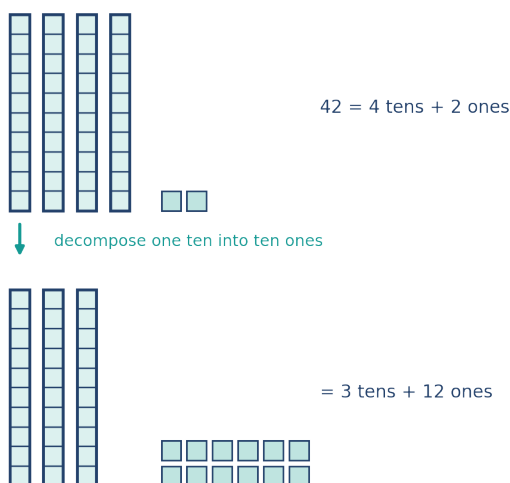


Figure 1. Before: crossing out with no meaning attached. After, 42 as four units of ten and two units of one, with one unit of ten decomposed to make the subtraction possible.

The takeaway: a student who has seen the decomposition can rebuild the written step after forgetting it. A student who has only seen the crossings-out cannot, because there is nothing to rebuild it from.

Worked Examples

Each example pairs one model students should reason with and the words to say alongside it. The procedures differ, but the mathematical actions underneath them are connected.

Example 1 · $42 - 17$ with base-ten units

42 renamed as 3 tens and 12 ones, then take away 17

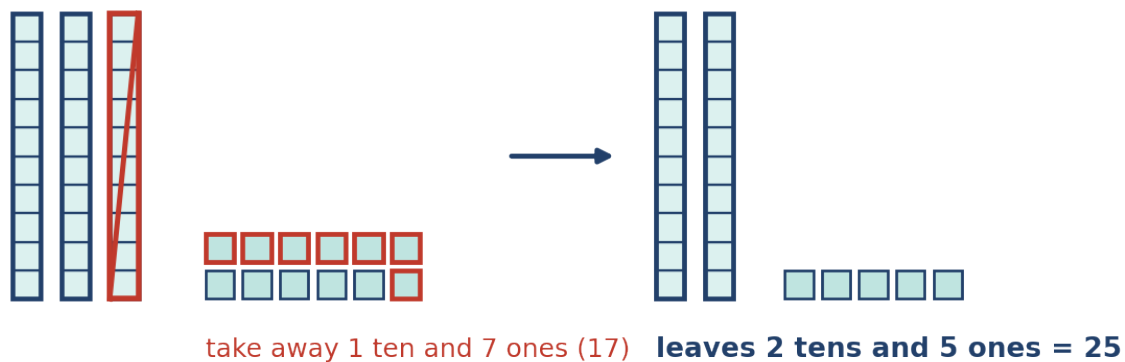


Figure 2. 42 is renamed as 3 tens and 12 ones, so 17 can be subtracted. The quantity does not change.

One unit of ten is decomposed into ten units of one. The representation changes, but the quantity remains equivalent, so $42 = 4 \text{ tens} + 2 \text{ ones} = 3 \text{ tens} + 12 \text{ ones}$. The written regrouping step records that decomposition.

- “What unit are we decomposing? How many units of one are in one unit of ten?”
- “Did the total change when we renamed 42 as 3 tens and 12 ones? How do you know?”
- “Where do you see this decomposition in the written algorithm?”

Example 2 · $\frac{1}{2} + \frac{1}{3}$ on a bar model

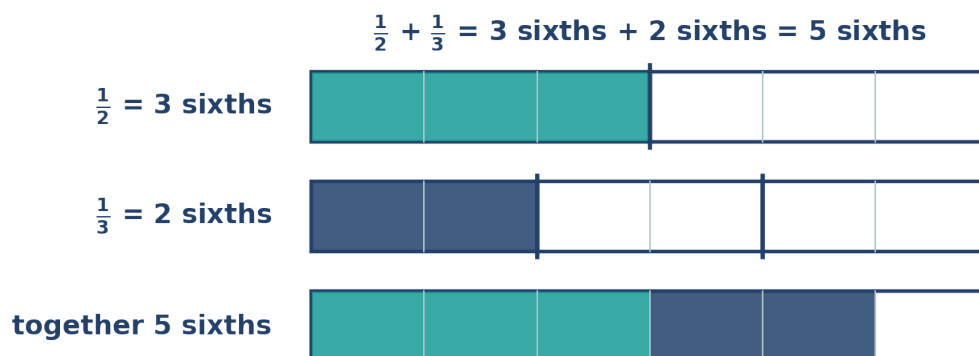


Figure 3. Halves and thirds renamed as sixths on the same unit of one: $\frac{1}{2} = 3 \text{ sixths}$, $\frac{1}{3} = 2 \text{ sixths}$, together 5 sixths.

Both fractions are partitioned into sixths on the same unit of one. The common denominator is a decision about the unit, not a rule. The structural sequence is partition, rename equivalently, then compose.

- “Halves and thirds name different-sized units. To add them, we write both using the same-sized unit. What unit works?”
- “Show me the halves and the thirds in sixths: three sixths and two sixths.”
- “Three sixths and two sixths make five sixths. The rule you learned says the same thing.”

Example 3 · 4×26 on an area model

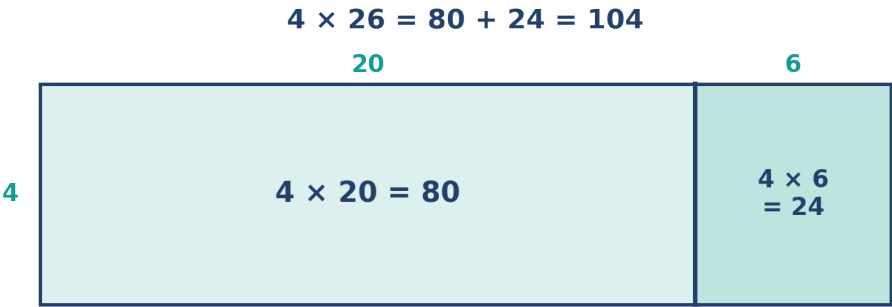


Figure 4. 4×26 as four iterations of 20 and 6: $4 \times 20 = 80$ and $4 \times 6 = 24$, composed to 104.

The 26 is partitioned into 20 and 6, then each part is iterated four times. The partial products are visible before they are written. The structural sequence is partition, iterate, then compose.

- “I partitioned the 26 into 20 and 6. Why is that allowed?”
- “Four twenties is 80 and four sixes is 24. Where is each one in the rectangle?”
- “The standard algorithm writes 80 and 24 too. Find the partitioning, iteration, and composing in your written work.”

One Move in Three Places

Example	Structural action	What the procedure records
42 – 17	Decompose and rename	Regrouping
$\frac{1}{2} + \frac{1}{3}$	Partition and rename equivalently	Common denominator
4×26	Partition, iterate, and compose	Partial products and standard algorithm

The procedures are different. The mathematical actions underneath them are connected.

The Language Shift

Keep this by the board. The left column asks for the step. The middle column asks for the idea the step records, and the right column names it.

Instead of	Say	Mathematical idea
Borrow from the 4.	Decompose one unit of ten into ten units of one.	Decompose and rename
Carry the 1.	Compose ten units of one into one unit of ten.	Compose a unit
Find the common denominator.	What partition holds both fractions?	Choosing the unit
Remember the rule.	What is the rule keeping track of?	The idea the rule records

Instead of	Say	Mathematical idea
Did you get 25?	How do you know 25 is reasonable?	Reasonableness first
Just do the steps.	Show me why that step works, then do it.	Explain, then perform.
You need more practice.	Let us go back to the model and rebuild the step.	Rebuild from the model.

What to Look For in a Classroom

- *Students explaining why a step works, not only that it answers.*
- *The idea and a model meeting students before the written procedure does.*
- *A reasonableness check before an accuracy check.*
- *Students pointing to where a written step appears in the model.*

Across the Grades

Putting understanding first looks different at each grade, but the move is the same every time: build the idea, then let the procedure grow from it.

Grade band	Model	The shift it asks for
Grades K-2	Bar model, number line	From counting individual units to composing and decomposing units.
Grades 3-5	Area model, number line	From carrying out an algorithm to seeing the unit structure and relationships the algorithm records.
Grades 6-8	Double number line, ratio table	From applying a rule to reasoning about the multiplicative or algebraic relationship the rule represents.

Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at mathsuccess.io/dmtinsights. Pick a procedure your students perform well this week, and ask a few of them why it works. Their explanations will tell you how much understanding sits under the fluency. Understanding gives a forgotten step somewhere to be rebuilt from.